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THE ROLE OF CONJUGATE CONVECTION IN MAGMATIC HEAT AND MASS TRANSFER

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ABSTRACT

The crystallization history of magma chambers is intimately coupled to the heat transfer systematics of the surrounding wall rock. In this paper we investigate the thermal interaction between magmatic and hydrothermal convection. Two models of magmatic convection are addressed: compositionally and thermally dominated flow both driven by heat loss to a hydrothermal system. For compositionally driven flow in a porous medium the temperature along the wall rock-magma interface is found to be a weighted constant. For thermally driven flow temperatures are found to increase upward along the intrusive contact The steady state thickness of the solid grown into the magma chamber was found to be O(10m) for siliceous melts and O(.01m) for basic melts, the difference in these values being a direct result of their differing viscosities. These models suggest that hydrothermal circulation can effectively quench the interface at the intrusive contact, particularily for siliceous magmas. The high temperatures recorded in the hydrothermally altered country rock near intrusions either record a short lived transient condition or the passage of fluids which have been in intimate contact with the magma.

1. INTRODUCTION

The interpretion of the diverse geochemical and textural character of igneous rocks has motivated earth scientists to investigate the complex interactions of crystallizing multi-component and multi-phase melts. A growing appreciation of the role of dynamics in crystallizing systems has produced studies whose focus extends past the traditional framework of equilibrium thermostatics to address the role of transport in the evolution of silicate melts. These studies employ laboratory, analytical and numerical models to address melt behavior and are often purposefully simplified to better isolate and reveal the physical processes thought to dominate in dynamic magmatic systems. To extend these models to a discussion of crystallization in magmas, the appropriateness of the assumptions used in the simplified models must be assessed. Of the many factors controlling the evolution of a crystal mush, the form of the thermal boundary conditions is important [1] as the dynamics of phase segregation in a partially molten system depends strongly on the method and efficiency of the overall thermal exchange.

A wealth of geologic evidence from studies of ore deposits and geothermal systems suggests that fluids play an important role in the evolution of magmatic systems. It is not difficult to appreciate that the entropy flux which sustains fluid structures in magmas [2, 3] depends on the evolution and disposition of the thermal regime of the wall rock [4]. This study addresses the thermal exchange between an open, crystallizing magma chamber and hydrothermal convection in a permeable host rock. The setting is that of two steady naturally convecting systems driven by their mutual thermal communication, hence the term 'conjugate' [5]. We desire the appropriate form of the thermal boundary condition at the interface between the two systems. The utility of finding the form of temperatures at the interface is that it then permits a decoupling of the two flow regimes through the boundary conditions while implicitly retaining the conjugate character of the magmatic setting.

Previous modeling of the interaction between hydrothermal circulation and cooling magma has been largely motivated by the search for mineral and geothermal resources. The works of Cathles [6], Norton and Knight [7] and Torrance and Sheu [8] employ numerical simulations to investigate hydrothermal regimes around conductively cooling plutons. Although these works provide a global view of the thermal and flow fields, they do not reveal the specific thermal character at the interface between the two systems. Analytical studies of the boundary layer flows at the magma-wallrock interface [9-17] have not considered the influence of thermal feedback in the modulation of flow structure. The study of Carrigan [18] investigates the balance of heat flux in the propagation of a cracking front into a cooling magma. In the engineering setting, conjugate convection has been studied by Bejan and Anderson [5], Sparrow and Prakash [19] and Viskanta and Lankford [20]; the quantitative techniques employed in these studies provided direction for this work.

2. PHYSICAL MODELS AND PROBLEM FORMULATION

Here we consider two models of conjugate convection between a magma and the surrounding saturated country rock into which it has intruded. Both models incorporate boundary layer fluid flow adjacent to a vertical wall where crystallization is occurring. The models were constructed to address two end member types of convection in the magma: compositionally and thermally driven flow. In the first model, the fluid motion in the magma chamber is driven by density changes that result from crystallization at the wall and that act in opposition to thermal effects on density. This yields an upward flow of relatively cooler but

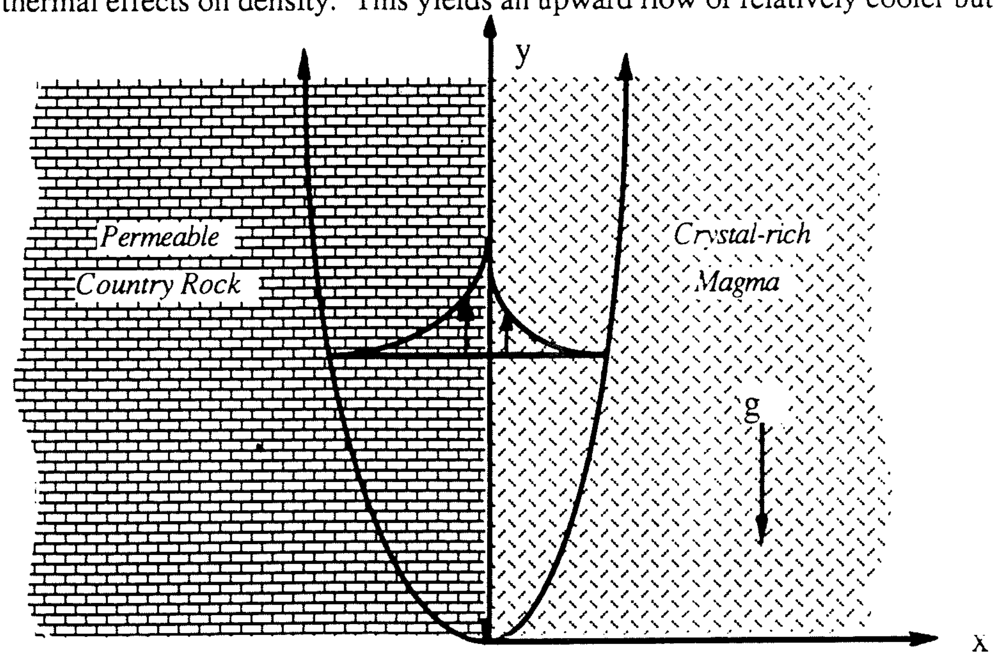

Figure 1. Schematic diagram of parallel flow conjugate convection in a porous medium. Sub-vertical lines coming up from the origin delineate the horizontal extant of the boundary layer flow. Arrows indicate relative magnitude of velocity for a convective efficiency ratio of O(1). Note that for porous media flow the thermal and velocity boundary layers are coincident as given by (4.14).

less dense magma. The second model considers magmatic boundary layer convection where downward flow is driven by the decrease in temperature across the magmatic thermal gradient. The thermal sink for both models is the hydrothermal convection in the adjacent country rock. In addition, both models invoke constant temperatures far from the contact and steady state flow.

The first model we consider is that where both the wall rock and the magma are porous semi-infinite media as shown in Fig. 1. The entire magma need not be a porous medium, rather only that portion near the wall where flow occurs. The two regions communicate thermally across an impermeable partition of negligible thermal resistance. This model assumes that the magma has little superheat and that thermodynamic equilibrium is maintained everywhere across the thermal gradient. The equilibrium assumption is an implicit statement of Gibb's phase rule: only one of temperature and concentration need to be specified in the expression for fluid density. The expansion coefficient appearing in that expression represents the net effect of the combined influence of temperature and concentration on density. For the first model the selective removal of components from the melt due to crystallization has the overall effect of reducing melt density. This yields an upward flow across the entire magmatic thermal gradient and vertical boundary layers obtain [21, 22]. We note that the model shown in Fig. 1 has a fundamentally different fluid structure than that in the porous counterflow model of Lowell [13] which requires some superheat or the assumption of steady-state disequilibrium.

The compositionally driven flow invoked here requires that crystallization occur all across the thermal gradient in the magma. For a multi-component system, a porous zone of crystals and melt will develop [23]. In addition to crystals growing directly from the melt, the porous framework could also have contributions from refractory components and the residuum from stoped blocks. As the magma near the wall continues to crystallize, the permeability at any point in the magma will decrease, eventually yielding a solid region at the contact. However, if thermal stresses induce fracturing in the just formed solid [24 -26], the system depicted in Fig. 1 could propagate laterally in a steady state fashion as suggested by Carrigan [18]. The quantitative description developed in Appendix 1 should be considered to be in a Lagrangian form with respect to this translation of the system.

A quantitative description of this parallel flow system is developed in Appendix 1. It is found that the thermal efficiency of the hydrothermal convection relative to the magmatic convection can be expressed as

$$\gamma = \frac{k' \, Ra'^{1/2}_{k}}{k \, Ra^{1/2}_{k}} \tag{2.1}$$

where the nomenclature is defined in Table 1.

For large values of γ the thermal efficiency of the convection in the wallrock is superior to that in the magma and the thermal character of the interface between the two systems will be like that of the country rock background value. This would yield a wide, low temperature flow regime in the wallrock. Conversely, if γ is small, magmatic convection dominates the thermal exchange producing a small temperature drop between the interface and the center of the magma chamber and a narrow, high temperature thermal aureole would develop in the country rock adjacent to the intrusion. The value of γ for a range of geologic parameters can be determined from Table 1. Except for a low viscosity basalt, γ is $O(10^3)$ and the interface temperature will be very close to that of the background.

One of the goals of this study is to determine the functional form of the interface temperature. The dimensional interface temperature for parallel conjugate natural convection is

$$T_w^* = \frac{(T_m^* - T_c^*)}{\gamma^{2/3} + 1} + T_c^* \tag{2.2}$$

Temperatures along the interface are found to be a constant weighted by the thermal efficiency parameter γ. We find in (2.2) that the wall temperature does not depend on the height up the wall. The dependence of the wall temperature on the vertical coordinate is contained in the Rayleigh numbers appearing in the thermal efficiency parameter. As the

Rayleigh numbers appear in a ratio both raised to the same power, the vertical coordinate drops out.

These results suggest that laboratory and quantitative models that address systems similar to that shown in Fig. 1 are justified in using a constant temperature boundary condition at the wall. This is particularly applicable to laboratory models of crystallization where a refrigerant is circulated in the walls of the experimental apparatus to enhance crystallization kinetics.

The second model we wish to consider is shown in Fig. 2. Heat from the magma drives hydrothermal flow in the country rock as before, however the flow regime in the magma is quite different. Instead of developing a permeable mushy zone of crystals, it is assumed that the porosity in the crystal zone is small and flow in the magma occurs outside of the region of crystal growth. Another difference between this model and the first is that any density changes due to crystallization act in concert with the increase in density from cooling. This yields downward flow in the magma and the conjugate flow now has an opposing structure. This model serves as an end member to the first as it addresses thermally driven flow in a free fluid, rather than compositionally driven flow in a porous media. This model is much like that addressed by Spera et al. [15].

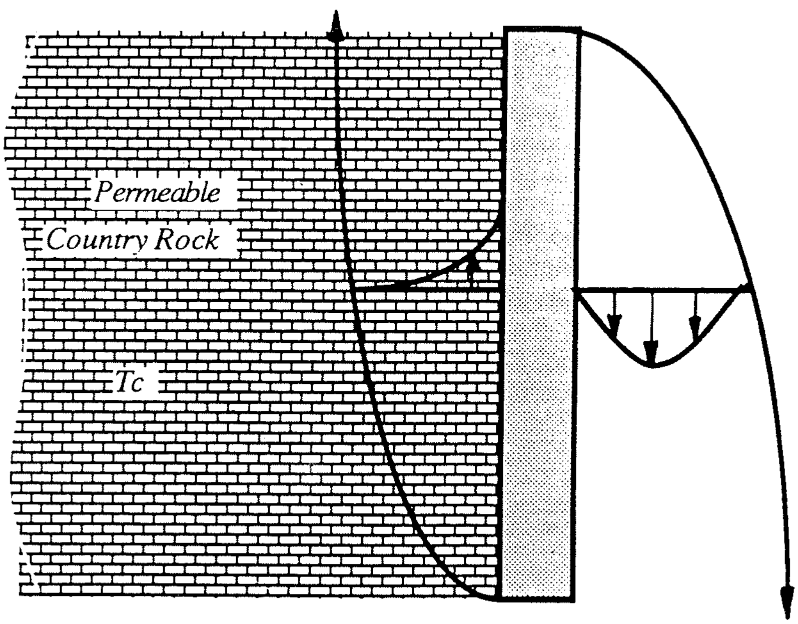

Figure 2. Schematic diagram of counterflowing conjugate convection. Subvertical lines are margins of boundary layers as in Fig. 1. Dot pattern is solid wall grown into magma chamber. Actual wall thickness will vary with distance along the wall as in (2.6).

The region with the dot pattern in Fig. 2 represents a solid grown into the magma chamber from the contact between the country rock and the magma. The temperature of the wall facing the magma chamber side is governed by the phase diagram of the magma and is presumed to be near the eutectic value. The temperatures along the other side of the wall, which represents the original intrusive contact, can be found by equating the heat flux from the magma to the flux across the country rock convective boundary layer. This yields an expression for temperatures along the original intrusive contact

$$T_w^* = T_c^* + \left(\frac{k}{k'}\right)^{2/3} \frac{\left(\frac{g\beta}{\alpha\upsilon}\right)^{1/6}}{\left(\frac{gK\beta'}{\alpha\upsilon}\right)^{1/3}} \left(T_m^* - T_s^*\right)^{5/6} \frac{y^{1/3}}{(H-y)^{1/6}} \tag{2.3}$$

TABLE 1. Nomenclature and Thermal Property Data

a	viscosity coefficient		.03 1/K
C_p	specific heat capacity as defined in Appendix 1		4186 J/kg K
h	heat transfer coefficient		
H	height of magma chamber		3 km
k	conductivity	water-rock matrix	2.5 W/m K
		magma-crystal matrix	2.0 W/m K
K	permeability	country rock	10^{-12} - 10^{-17} m^2
		magma	10^{-8} - 10^{-12} m^2
L	latent heat	magma	4.0 x 10^5 J/kg
n	power law viscosity exponent	magma	.867
P	pressure		
r	radius of magma chamber		2 km
Ra_K	porous medium Rayleigh number		$Ra_K = (Kg\beta\Delta Ty) / (\alpha\upsilon)$
T	temperature	magma	850 ^oC - 1050 ^oC
u, v	horizontal and vertical velocity		
x, y	cartesian coordinates, y positive upward		
X	wall thickness		
α	thermal diffusivity	magma	8.0 x 10^{-5} m^2/s
		water-rock matrix	5.0 x 10^{-7} m^2/s
β	thermal expansion coefficient	magma (model 1)	- 5.0 x 10^{-4} 1/K
		magma (model 2)	2.5 x 10^{-5} 1/K
		water	4.0 x 10^{-4} 1/K
μ	dynamic viscosity	magma	10 - 10^8 kg/m s
		water	4.0 x 10^{-4} - 1 kg/m s
υ	kinematic viscosity		
ρ	density	magma	2300 - 3000 kg/m^3
		water	960 - 1000 kg/m^3
δ	boundary layer thickness		
ϕ	porosity		
χ	power law viscosity coefficient	magma	845 N s^n $/m^2$
σ	shear stress		
γ	thermal efficiency parameter		
λ, ζ	roots of equations for temperature and velocity (Appendix 1)		

Superscripts:

( )'	quantities on country rock side
( )*	dimensional quantities

Subscripts:

A	average value
c	country rock value
m	magma value
o	reference value
s	solid wall quantities (second model)
w	value at wall

This expression is only valid away from the point $y = H$ where the boundary layer singularity produces an infinite thermal flux.

The parameter group in the second term on the right side of (2.3) gives a measure of the relative thermal efficiency of the conjugate convection

$$\frac{1}{\gamma(y)} = \left(\frac{k}{k}\right)^{2/3} \frac{\left(\frac{g\beta}{\alpha\upsilon}\right)^{1/6}}{\left(\frac{gK\beta}{\alpha\upsilon}\right)^{1/3}} \frac{y^{1/3}}{(H - y)^{1/6}} \tag{2.4}$$

For (2.3) to be valid T_w^* must be less than T_s^*. This is equivalent to imposing the following condition

$$\frac{1 - \frac{T_c^*}{T_s^*}}{\frac{(T_m^* - T_s^*)^{5/6}}{T_s^*}} \leq \gamma(y) \tag{2.5}$$

If this inequality is violated melting could occur at the wall and the steady state assumption will no longer be valid. This requirement is overlooked by Carrigan [18] who suggests that magmatic convection is more efficient than hydrothermal convection. If this were true the thermal flux from the magmatic convection would cause a build up of heat in tne wall between the magmatic and hydrothermal boundary layers. This in turn would lead to melting not cooling. Hence, magmatic convection cannot be more efficient than convection in the country rock if the steady state assumptions employed here and by Carrigan [18] are to hold. Work in progress suggests that (2.5) is often satisfied as the variation in fluid properties, particularily viscosity, strongly inhibts convection ahead of a crystallizing front.

In steady state the thermal flux across each boundary layer and the solid wall must be equal. This permits an expression for the wall thickness to be found. The wall will grow into the melt until the thermal resistance of the wall equals that of the magmatic boundary layer [27, 28]. The wall thickness is given by

$$X(y) = \frac{k_s(T_s^* - T_w^*)}{h_m(y)\,(T_m^* - T_s^*)} = \frac{k_s\,(T_s^* - T_w^*)\,(H - y)}{k\,Ra_m^{1/4}\,(T_m^* - T_s^*)} \tag{2.6}$$

where

$$Ra_m = \frac{g\beta\,(T_m^* - T_s^*)\,(H - y)^3}{\alpha\upsilon} \tag{2.7}$$

For small amounts of superheat in the magma, the temperatures along the country rock contact will be relatively low. This is due to the decreased thermal flux through the magma which permits the solid wall growing from the melt to become very wide. The overall thermal gradient between the country rock and magma reservoir temperatures will then reside in the wall.

The parameter that has the most influence on the steady state thickness is the viscosity. A high viscosity in the magma results in the less efficient delivery of heat by the magma to the wall. This in turn allows the wall to grow relatively wide. Conversely, if the magma has a low viscosity the thermal flux to the wall will be enhanced, resulting in a relatively thin wall. Using the viscosity relationship for siliceous melts proposed by Spera et al. [15]

$$\mu = \mu_o \exp[a(T_m^* - T_s^*)]$$

and other values in Table 1 we find the average value of the wall thickness based on the average Nusselt number to be of O(10m). Hardee [29] and Hardee and Dunn [30] suggest a power law viscosity relationship for basaltic melts of the form

$$\sigma = \chi \left(\frac{\partial u}{\partial x}\right)^n$$

which yields a steady-state thickness O(.01m).

Invoking steady temperatures far from the wall both in the country rock and the magma implies steady-state open system behavior. The required mass flow rate to maintain steady-state can be calculated by equating the heat loss from the magma chamber to the sensible and latent heats of an underplating basalt [15]. The heat loss per unit time (power) out of a magma chamber with radius r that is being cooled by hydrothermal convection at the sides is

$$Q = \frac{\pi r}{H^{1/2}} \int_0^H \frac{k\, Ra_K^{1/2}\, (T_w^* - T_c^*)^{3/2}}{(T_m^* - T_c^*)^{1/2}\, y^{*1/2}}\, dy^*$$

The mass flow rate of new magma needed to offset the convective heat loss is

$$\dot{m} = 2\pi r k \frac{(T_w^* - T_c^*)^{3/2}}{(T_m^* - T_c^*)^{1/2}}\, Ra_K^{1/2} \bullet \frac{1}{C_p\, \Delta T^*} \qquad (2.8)$$

where the heat capacity term is defined to include the effects of latent heat as in Appendix 1. The required mass flow is O(10^{11}g/yr). Although this figure is compatible with infusion rates estimated for basaltic volcanic centers such as Kilauea [15] it is not certain that it is reasonable in the continental setting where volcanism is more distributed.

Before closing a quantitative discussion of the two models, we note that for the present analysis to apply the magma chamber aspect ratio is constrained by

$$\left(\frac{H}{2r}\right)^{1/2} << Ra_K^{1/2}$$

for the first model and

$$\left(\frac{H}{2r}\right)^{1/4} << Ra^{1/4}$$

for the second model where Ra is defined by (2.7).

3. CONCLUSIONS

In order to determine the appropriate form of the boundary conditions between a convecting magma and a hydrothermal regime, expressions for the temperature along the interface have been derived for two end member geometries. For the first model it is found that parallel conjugate convection yields a constant wall temperature weighted by the thermal efficiences of the two boundary layer flows. For counter-flowing conjugate convection where a solid has grown into the melt the temperature was found to increase upward along the wall rock contact in the manner given by (2.3).

The thermal efficiency parameter provides a measure of the overall entropy exchange of hydrothermal convection relative to magmatic convection when the two are coupled in conjugate flow. For the models considered here the thermal efficiency can range from O(1) to quite large values. To be consistent with the assumptions employed in the models the thermal efficiency parameter should not drop below one. Fluid structure such as boundary layer flow can exist only by virtue of the entropy flux out of the system [2]. If the thermal efficiency parameter is less than one the entropy flux into the wall rock will be diminished, resulting in degradation of the boundary layer structure in the magma or melting of the wall rock. Thus the thermal efficiency of boundary layer flow in the magma chamber is not likely to exceed that in the country rock. This is supported by field evidence: large scale fusion textures are not common at the margins of intrusions.

The large differences in the steady-state thickness of the wall grown from the acid and basic melts can be directly attributed to their extreme viscosity contrasts. This reinforces arguments by Spera et al. [15] as to the importance of invoking the appropriate viscosity relationship in modeling magmatic convection. In the limiting case of a very high viscosity the Rayleigh number for the magma becomes very small and the boundary layer assumptions no longer hold. In this limit a model based on pure conduction in the magma is more appropriate such as that in [31].

Changes in the thermal efficiency of the wall rock convection will change the thickness and physical character of the wall that has grown from the melt. If the permeability of the wall rock increases due to cracking from thermal stresses [8, 18, 24-26, 32, 33] the solid will increase in thickness. If the thermal efficiency of the wall rock decreases due to the deposition of hydrothermal minerals the wall will decrease in thickness. This would lead to a disaggregation of the wall and perhaps distribute crystals to other portions of the magma chamber where disequilibrium textures could develop.

The open system character of the magmatic setting makes it difficult to test the models presented here. Eruption and over printing from crystallization post-emplacement deformation make it unlikely that relic fluid structures can be confidently recognized in exposed plutons and hypabyssal rocks. The fluid inclusion and oxygen isotope systematics of contact aureoles provide evidence for the circulation of fluids. However, it is difficult to resolve the timing of the circulation and the origin of the fluids. The presence of high temperature skarn deposits near the contacts suggests that either skarn formation occurred before hydrothermal circulation thermally swamped the interface or that skarn forming fluids have crossed the interface from the magma; the latter possibility is in agreement with the oxygen isotope data of Bowman et al. [34]. This work does demonstrate that appealing to the thermal coupling of the magma-wall rock system can help to constrain studies of contact relations based on equilibrium thermostatics.

4. APPENDIX 1: ANALYSIS OF CONJUGATE NATURAL CONVECTION IN POROUS MEDIUM

Here we develop a quantitative description of the first model which is shown in Fig. 1. Fluid structures on both sides of the wall are governed by a system of equations of the form

$$\frac{\partial u^*}{\partial x^*} + \frac{\partial v^*}{\partial y^*} = 0 \tag{4.1}$$

$$u^* = -\frac{K}{\mu}\frac{\partial P^*}{\partial x^*} \tag{4.2}$$

$$v^* = -\frac{K}{\mu}\left(\frac{\partial P^*}{\partial y^*} + \rho^* g\right) \tag{4.3}$$

$$u^*\frac{\partial T^*}{\partial x^*} + v^*\frac{\partial T^*}{\partial y^*} = a\left(\frac{\partial^2 T^*}{\partial x^{*2}} + \frac{\partial^2 T^*}{\partial y^{*2}}\right) \tag{4.4}$$

$$\rho^* = \rho_o^*(1 - \beta(T^* - T_o^*)) \tag{4.5}$$

Take cross derivatives of (4.2) and (4.3) and subtracting

$$\frac{\partial u^*}{\partial y^*} - \frac{\partial v^*}{\partial x^*} = g\beta \frac{\partial T^*}{\partial x^*} \tag{4.6}$$

Next define the dimensionless variables:

$$x = x^*/\delta, \qquad y = y^*/H, \qquad T = \frac{T^* - T_c^*}{T_m^* - T_c^*}$$

$$u = u^*\delta/\alpha, \qquad v = v^*\delta^2/\alpha H$$

Latent heat that is realeased by magma crystallizing as it percolates through the porous crystal matrix is treated by defining the specific heat as

$$C_p = C_p^* + \frac{L}{\Delta T^*} \tag{4.7}$$

Scaling arguments [35] show that

$$\delta \sim H/Ra_k^{1/2}$$

where we invoke suitability chosen average values for the permeability and kinematic viscosity. The influence of variable viscosity on porous media flow has been briefly considered by Weber [35] and Bergantz [7]. It was found that a temperature dependent viscosity yielded a wider thermal boundary layer in the cooled region and a narrower boundary layer in the warmed region. However, this stretching of the boundary layer does not alter the fundamental structure of the boundary layer flow. For purposes of assessing the heat transfer portion of the problem, an appropriate average will serve. Nilsen et al. [17] reached a similiar conclusion regarding double-diffusive flow in a free fluid.

After invoking the usual boundary layer and Boussinesq approximations (4.1) thru (4.4) become

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0 \tag{4.8}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{\partial^2 T}{\partial x^2} \tag{4.9}$$

and for the country rock

$$\frac{\partial v}{\partial x} = \frac{\partial T}{\partial x} \tag{4.10}$$

for the magma:

$$\frac{\partial v}{\partial x} = -\frac{\partial T}{\partial x} \tag{4.11}$$

with boundary conditions:

country rock [0, $-\infty$]

$x \rightarrow -\infty$, $T = 0$, $v = 0$ (4.12)

$x \to 0$, $T = T_w$, $u = 0$

magma chamber $[0, +\infty]$:

$$x \to +\infty,\ T = 1,\ v = 0 \qquad (4.13)$$

$$x \to 0,\ T = T_w,\ u = 0$$

Darcy's law neglects viscous forces and a no-slip velocity boundary condition is not required at the interface. Hsu and Cheng [36] find that the slip condition introduces negligible error if

$$K\sqrt{\frac{\rho_0 g\beta\Delta T}{\phi\mu\alpha y}} << 1$$

For the range of permeabilities considered here, the slip condition introduces no significant error.

First we consider the country rock side $[0, -\infty]$, integrate (4.10) and using (4.12a)

$$v = T \qquad (4.14)$$

Next linearize energy equation (4.9) by invoking the Oseen technique [5, 37]. This is done by replacing the horizontal velocity and vertical temperature derivative by their averages in x

$$u_A(y) = \frac{1}{\delta(y)}\int_0^{\delta(y)} u(x,y)\,dx \qquad \frac{\partial T}{\partial y_A}(y) = \frac{1}{\delta(y)}\int_0^{\delta(y)} \frac{\partial T}{\partial y}(x,y)\,dx$$

These are now substituted into (4.9) and using (4.14)

$$\frac{\partial^2 v}{\partial x^2} - u_A\frac{\partial v}{\partial x} - \frac{\partial T}{\partial y_A}v = 0 \qquad (4.15)$$

which is a constant coefficient ordinary differential equation in x with the general solution

$$v = \sum_{i=1}^{2} C_i e^{\zeta_i x}$$

where

$$\zeta_{1,2} = \frac{1}{2}\left(u_A \pm \sqrt{u_A^2 + 4\frac{\partial T}{\partial y_A}} \right)$$

The vertical derivative of temperature is positive for all x and ζ_1 and ζ_2 must have opposite signs. Discard the negative root to keep the solution bounded which yields

$$v = T_w e^{\zeta x} = T \qquad (4.16)$$

It is required that the solution of (4.16) satisfy the energy equation integrated over $[0, -\infty]$:

$$(uT)_0^{-\infty} + \frac{d}{dy}\int_0^{-\infty} vT\,dx = \left(\frac{\partial T}{\partial x}\right)_0^{-\infty}$$

which leads to

$$\frac{d}{dy}\left(\frac{T_w^2}{2\zeta}\right) = \zeta\, T_w \tag{4.17}$$

A similiar analysis for the magma chamber side yields the following equations

$$v = (1 - T_w)\, e^{-\lambda x} \tag{4.18}$$

$$T = 1 - (1 - T_w)\, e^{-\lambda x} \tag{4.19}$$

$$\frac{d}{dy}\left[\frac{(1 - T_w)^2}{2\lambda}\right] = \lambda\, (1 - T_w)$$

To solve this system of equations we require an expression for T_w. To this end scaling arguments and numerical experiments reveal that T_w is constant in y. The resulting expressions for ζ and λ are

$$\zeta = \frac{\sqrt{T_w}}{2\sqrt{y}} \qquad \lambda = \frac{\sqrt{(1 - T_w)}}{2\sqrt{y}}$$

To find the specific form of T_w we invoke the continuity of heat flux condition at the interface whose nondimensional form is

$$\gamma\left(\frac{\partial T}{\partial x}\right)_{x = 0^-} = \left(\frac{\partial T}{\partial x}\right)_{x = 0^+}$$

where

$$\gamma = \frac{k' \, Ra_K'^{\,1/2}}{k\, Ra_K^{1/2}} \tag{4.20}$$

is the dimensionless ratio that expresses the relative thermal efficiency of the conjugate flow. In terms of ζ and λ

$$\gamma\frac{\zeta}{\lambda} = \frac{(1 - T_w)}{T_w}$$

$$T_w = \frac{1}{\gamma^{2/3} + 1} \tag{4.21}$$

$1/\zeta$ and $1/\lambda$ are equal to the boundary layer thicknesses in the country rock and the magma chamber respectively.

The Nusselt number fot the porous conjugate convective system shown in Fig. 1 is

$$Nu_y = \frac{(-k \frac{\partial T^*}{\partial x^*})_{x=0}\, y}{(T_w^* - T_c^*)\, k} = \frac{1}{2} Ra_K^{1/2} \qquad (4.22)$$

where the Rayleigh number is now defined as

$$Ra_K = \frac{gK\beta(T_w^* - T_c^*)y}{\alpha\upsilon}$$

A test of the error introduced by the Oseen linearization technique used above can be done by comparing these results with a more exact analysis. Cheng and Minkowycz [10] employ the similarity variable technique to resolve the convective flow about a vertical isothermal wall in a porous medium. They find

$$Nu_y = .444\, Ra_K^{1/2}$$

The Nusselt number obtained here with the Oseen technique is in good agreement with that found in [10].

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