

MELT STABILITY AND COMPACTION IN A PARTIALLY MOLTEN SILICATE
LAYER HEATED FROM BELOW

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ABSTRACT

In this paper, the possibility of convective or Rayleigh-Taylor instability in a growing layer of partially molten material, heated from below, is investigated. The relative importance of matrix compaction and thermal conduction on the dynamics of the partially molten layer is also studied by a scaling analysis of the appropriate dimensionless equations. The analysis shows that, for low viscosity melts, thermal conduction is unimportant; and melt dynamics is controlled by a combination of matrix compaction and buoyantly-driven instabilities within the melt itself. Once these instabilities occur, melt migration becomes a two- or three-dimensional process, and the chemical composition of the melt that leaves the source zone represents a mixture of melts produced at different temperature-pressure regimes in the source area. In melts with high viscosity, thermal conduction dominates the thermal regimes. Compaction and shear deformation of the matrix must both act to segregate the melt from the matrix. The melt is stable with respect to thermal or compositional convection until a large quantity of liquid has coalesced.

1. INTRODUCTION

When a rock composed of silicate minerals begins to melt, either as a result of pressure release melting as in an ascending diapir, or as a result of heating a stationary layer from below, the melt fraction tends to concentrate along grain boundaries. An interconnected network of melt channels develops, probably after melt fraction exceeds a few percent [1,2], so that the partially molten system behaves as a permeable medium. The melt plays the role of the pore fluid and the unmelted rock provides the matrix through which the melt flows. The permeability of the partially molten system must be determined from

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assumptions regarding the shape of the channels, the grain size, and the porosity. The permeability-porosity relationship is usually expressed in a form similar to the well-known Carmen-Kozeny relationship [3], for want of any direct data.

A mathematical description of the dynamics of partially molten silicate rock as a porous-permeable medium is considerably more difficult than that for fluid-saturated rock such as is encountered in hydrology for several reasons. In the hydrological problem, the rock is under lithostatic pressure, and the fluid is generally under hydrostatic pressure. The rock matrix is considered to be rigid and undeformable; consequently, there is no tendency for the fluid to migrate relative to the rock. In the case of partially molten rock in the earth's interior, however, the rock matrix is assumed to be deformable. Pressure differences between the matrix and melt lead to a deformation of the matrix and migration of the melt relative to the matrix. The deformation of the matrix can be viewed, in some cases, as a simple compaction, and the expulsion of the melt from the matrix is analogous to the process of squeezing water from a sponge. A further complication that arises in the partial melt situation is that, as the melt migrates through the matrix, thermodynamic equilibrium is assumed to be maintained. The assumption of thermodynamic equilibrium implies that local melting or freezing occurs as the melt passes through the matrix, resulting in a transfer of material between the melt and matrix, together with an absorption or release of latent heat. Furthermore, the temperature and/or pressure gradients that occur in the partially molten system give rise to compositional gradients within the melt. The concurrence of temperature and compositional gradients leads to complicated dynamical effects in the melt that may affect the dynamics of melt migration. It is this possible interaction that will be explored semi-quantitatively in this paper.

The dynamics of a partially molten silicate rock as flow in a porous medium has been treated by several investigators [4-12] without regard to the recognition of the natural melt as a multicomponent fluid and that the melt and matrix composition at a given temperature and pressure may be fixed by assuming local thermodynamic equilibrium between the melt and the matrix. Calculations based on such models preclude the possibility of convective instability within the melt itself and, therefore, ignore the possible interaction between the melt migration and the convective instability. Furthermore, the results of these calculations provide little insight into the composition of silicate melts that may accumulate to form shallow magma chambers. Hills et al. [13] develop a formulation that in final form is similar to McKenzie's [7], but they do not carry out any subsequent calculations. Ribe [14] has developed a one-dimensional, steady-state model of melt migration of a two-component diapir that includes the assumption of local thermodynamic equilibrium. He does not consider, however, the possible role of convective instability within the

diapir. Finally, melt stability of a layer heated from without considering

In order to study and instability, and momentum for a two-component system of energy. In systems are used to matrix as function will focus on the develop in a part heated from below applied to an ascending point no detailed relative importance melt production and melt compositions a starting point melt migration in or non-isobaric nature

2. THE BASIC EQUATIONS

A two-component system conservation of mass are considered. The component in the

$$\frac{\partial}{\partial t}$$

$$\frac{\partial}{\partial t} (\phi)$$

$$\frac{\partial}{\partial t} ((1 - \phi))$$

$$\frac{\partial}{\partial t} ((1 - \phi) \rho_s)$$

where ϕ is the porosity are the melt (f) fractions of one melt and matrix, and 2 are transfer U_i are the velocities A complete list of Appendix. Equation of McKenzie [7] for

annels, the grain porosity relationship the well-known of any direct data.

ics of partially medium is considered saturated rock such as asons. In the hydrostatic pressure, and pressure. The rock ornable; conse- d to migrate relative ten rock in the is assumed to be the matrix and melt migration of the melt of the matrix can be ion, and the expulsion, and the process of complication that at, as the melt equilibrium is of thermodynamic freezing occurs as ting in a transfer of ether with an absorp- ore, the temperature he partially molten ts within the melt. itional gradients the melt that may t is this possible antitatively in this

icate rock as flow in al investigators [4- the natural melt as a d matrix composition e fixed by assuming he melt and the s preclude the possi- he melt itself and, on between the melt . Furthermore, the ttle insight into the ccumulate to form 3] develop a formula- (enzie's [7], but they ions. Ribe [14] has e model of melt migra- udes the assumption of s not consider, how- stability within the

diapir. Finally, Lowell [15] has considered the problem of melt stability of a two-component silicate partial melt in a layer heated from below and in a one-dimensional diapir, but without considering the role of melt migration.

In order to study the combined processes of melt migration and instability, the basic equations of conservation of mass and momentum for the melt and the matrix are written, assuming a two-component silicate system, and an equation for conservation of energy. Phase diagrams for relatively simple silicate systems are used to deduce the composition of the melt and matrix as functions of temperature and pressure. The analysis will focus on the melt migration and instability that may develop in a partially molten layer that grows in time as it is heated from below. With some modification, the analysis may be applied to an ascending, partially molten diapir. At this point no detailed calculations will be carried out. Rather the relative importance of compaction and melt instability during melt production and migration will be identified for a range of melt compositions. The analysis given here will thus serve as a starting point for more detailed mathematical modeling of melt migration in the earth's interior when the non-isothermal or non-isobaric nature of melt production is considered.

2. THE BASIC EQUATIONS

A two-component system and the appropriate equations for conservation of mass and momentum in the melt and in the matrix are considered. The equations of conservation of mass for each component in the melt and matrix according to Ribe [14] are:

$$\frac{\partial}{\partial t} (\phi \rho_f \chi_f) + \frac{\partial}{\partial x_i} (\phi \rho_f \chi_f u_i) = \Gamma_1 \quad (2.1)$$

$$\frac{\partial}{\partial t} (\phi \rho_f (1 - \chi_f)) + \frac{\partial}{\partial x_i} (\phi \rho_f (1 - \chi_f) u_i) = \Gamma_2 \quad (2.2)$$

$$\frac{\partial}{\partial t} ((1 - \phi) \rho_s \chi_s) + \frac{\partial}{\partial x_i} ((1 - \phi) \rho_s \chi_s U_i) = -\Gamma_1 \quad (2.3)$$

$$\frac{\partial}{\partial t} ((1 - \phi) \rho_s (1 - \chi_s)) + \frac{\partial}{\partial x_i} ((1 - \phi) \rho_s (1 - \chi_s) U_i) = -\Gamma_2, \quad (2.4)$$

where ϕ is the porosity (or volume fraction of melt), ρ_f and ρ_s are the melt (f) and matrix (s) density, χ_f and χ_s are the mass fractions of one component (e.g., the "light component") in the melt and matrix, Γ_1 and Γ_2 are the rates at which components 1 and 2 are transferred from the melt to the matrix, and u_i and U_i are the velocities of the melt and matrix, respectively. A complete list of symbols used in this paper are given in the Appendix. Equations (2.1) through (2.4) are analogous to those of McKenzie [7] for a one-component fluid. Addition of (2.1)

conservation of mass in
conservation of momentum
[7]:

$$\rho_f g \delta_{i3} \quad (2.5)$$

$$-\phi) \sigma_{ij}^i, \quad (2.6)$$

$$+ \zeta_s \delta_{ij} \frac{\partial U_l}{\partial x_l}. \quad (2.7)$$

pressure, η_s the shear
viscosity, and g the

in local thermal
at the same tempera-
duction and heating
is written, again

$$\phi \alpha_f) \partial P / \partial t$$

$$u_i \left(\frac{\partial T}{\partial x_i} - \frac{T \alpha_f}{\rho_f c_p} \frac{\partial P}{\partial x_i} \right)$$

$$\Gamma_1 + \Gamma_2) . \quad (2.8)$$

at constant pres-
ension, κ is the
xture, and L is the
expressions for the
ture (e.g., see Roy
ssion is the weighted
of data, it will be
heat of fusion for a
. $L = T \Delta S$, where ΔS
enerally assumed that
or silicic melts and
re interval ΔT over
melts and perhaps

~200°C for mafic melts. Thus L would increase by roughly 20% over the melting range. Such a variation is considered small when one considers the uncertainty in the other parameters. In the analysis, L will be assumed to be a constant. Equations (2.1) through (2.8) represent eleven equations for the fifteen variables u_i , U_i ($i=1,2,3$), P , T , ρ_f , ρ_s , ϕ , χ_s , χ_f , Γ_1 , Γ_2 . Thus, there are still four equations necessary to specify the system. These are equations of state for ρ_s and ρ_f of the form

$$\rho_s(P, T, \chi_s) = \rho_f(P, T, \chi_f) = 0 \quad (2.9)$$

and conditions expressing local thermodynamic equilibrium

$$\begin{aligned} \chi_f &= \chi_f(P, T) \\ \chi_s &= \chi_s(P, T) \end{aligned} \quad (2.10)$$

The equations (2.10) are determined from the pertinent phase diagrams.

3. A ONE-DIMENSIONAL MODEL--LAYER HEATED FROM BELOW

Equations (2.1) through (2.10) can be simplified considerably by considering one-dimensional models. The situation of a horizontal layer of rock heated from below is considered. This model is used mainly for the purpose of comparison with the compaction model of McKenzie [7] and Richter and McKenzie [8]. Physically, such a model might serve as a first approximation for the generation of high-silica melts formed in the earth's crust following the emplacement of more basic melts subjacent to the layer. Because the stability problem has been investigated by Lowell [15] for basaltic melts without consideration of magma migration, the focus of the ensuing development will be migration vis-a-vis instability. A range of melt compositions and transport properties will be considered.

Consider the one-dimensional transient problem. This problem is similar to that in Richter and McKenzie [8], except they ignored energy transport and phase equilibrium, and they assumed the layer thickness was fixed. In the current development an initially solid layer at the eutectic temperature T_e is assumed. At time $t = 0$ the base of the layer is raised to $T = T^*$ (where T^* is less than the liquidus). A partially molten layer begins to grow. For simplicity a perfect binary system is assumed. It will be assumed that the bulk composition is not at the eutectic, and that the bulk composition lies to the side of the eutectic such that $\chi_s = 0$. The analysis for the other side of the phase diagram ($\chi_f = 0$) would proceed in analogous fashion. Moreover, the Boussinesq approximation (i.e., $\rho_f = \rho_s$ everywhere except in the buoyancy terms in the momentum equation) is made. Then the following non-dimensional variables are invoked:

$$\phi' = \phi/\phi_0 ; \quad \phi_0 = \text{constant, the partial melt fraction at the phase change boundary} \quad (3.1)$$

$$K' = K/K_0 ; \quad K_0 = \phi_0^n \ell^2/d, \quad \ell \text{ is grain size, } d = \text{constant} \quad (3.2)$$

$$(w', W') = (w, W)/W_0 ; \quad W_0 = K_0/\eta_f (1-\phi_0) \Delta\rho g \quad (3.3)$$

$$z' = z/\delta ; \quad \delta = \left[\frac{K_0(\tau_s + 4/3\eta_s)}{\eta_f} \right]^{1/2} \quad (3.4)$$

$$t' = t/\tau ; \quad \tau = \phi_0 \delta / W_0 \quad (3.5)$$

$$T' = (T - T_e)/(T^* - T_e) \quad (3.6)$$

Upon substitution of (3.1) through (3.6) into (2.1) through (2.8) and making simplifications similar to Richter and McKenzie [8], one obtains:

$$\frac{\partial \phi T}{\partial t} - \frac{\partial}{\partial z} (1 - \phi_0 \phi) WT = 0 \quad (3.7)$$

$$\frac{d^2 W}{dz^2} - W/K - \frac{(1 - \phi_0 \phi)}{(1 - \phi_0)} = 0 \quad (3.8)$$

$$\frac{\partial T}{\partial t} = (\tau a / \delta^2) \frac{\partial^2 T}{\partial z^2} + S \left(-\frac{\partial \phi}{\partial t} + \frac{\partial}{\partial z} (1 - \phi_0 \phi) W \right) \quad (3.9)$$

$$w = - \frac{(1 - \phi_0 \phi) W}{\phi_0 \phi} \quad (3.10)$$

$$K = \phi^n, \quad (3.11)$$

where, for convenience, the primes have been dropped from the dimensionless variables. In equation (3.7) a phase relationship of the form $\chi_f = bT$ is incorporated, where b is a constant. In (19) it is assumed that $c_s^s = c_p^f = c_p$, $a = \kappa/\rho c_p$, and $S = L\phi_0/\Delta T c_p$. Equations (3.8), (3.10), and (3.11) are identical to corresponding equations of Richter and McKenzie [8]. Equation (3.9) is the energy equation that is neglected in Richter and McKenzie's [8] treatment; and (3.7) is analogous to their conservation of mass equation, except in the present analysis the mass fraction of the light component is conserved explicitly. Because the light component χ_f is related to the temperature through the requirement of thermodynamic equilibrium, equations (3.7) through (3.10) represent four equations in four unknowns, ϕ , w , W , T . The parameter b relating the

mass fraction of the light component through the temperature does not appear in the equilibrium reason is that (3.7) is the energy transfer during the phase change process is driven by the temperature gradient in the melt and the crystal is assumed to be constant. The composition of the melt is assumed to be constant. These equations are compositional and temperature dependent. The density distribution is assumed to be perturbed density of critical importance. The problem is solved subject to the

$$w = W =$$

$$\phi =$$

$$T =$$

and initial conditions

where $h(t)$ is the boundary condition. This boundary

Equations (3.7) through (3.11) define the problem of migration in a growing crystal from below, in which the system is in a steady state, perturbation within the melt is assumed to be negligible. The state can be solved in non-trivial cases. Conservation of mass and energy migration are assumed to be negligible. The thermal regime can be defined by the scale variables W_0 and δ . The dimensionless variables w and W are defined by comparing h to δ as a function of time. It depends upon the parameters $\Delta T = 100^\circ\text{C}$, $c_p = 1$, W_0 , δ , τ , and $a\tau/\delta$.

partial melt
melt boundary (3.1)

constant (3.2)

$(-\phi_0) \Delta \rho g$ (3.3)

$]^{1/2}$ (3.4)

(3.5)

(3.6)

into (2.1) through
Richter and

(3.7)

(3.8)

$(-\phi_0 \phi) W$ (3.9)

(3.10)

(3.11)

been dropped from the
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where b is a con-
 $= c_p$, $a = \kappa / \rho c_p$, and
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and McKenzie [8].
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mass fraction of the light component in the fluid to the temperature through the constraint of thermodynamic phase equilibrium does not appear in equations (3.7) through (3.11). The reason is that (3.7) through (3.11) involve heat and mass transfer during the compaction process and the dynamics of this process is driven by the bulk density difference between the melt and the crystalline matrix. This density difference is assumed to be constant. Superimposed on the compaction dynamics are compositional and/or thermal instabilities within the melt itself. These instabilities would arise because of compositional and temperature gradients that would affect the density distribution in the melt. In the calculation of this perturbed density distribution, the magnitude and sign of b are of critical importance. Equations (3.7) through (3.10) are solved subject to the following boundary conditions;

$$w = W = 0 \quad \text{at} \quad z = 0, \quad z = h(t)/\delta \quad (3.12)$$

$$\phi = 1 \quad \text{at} \quad z = h(t)/\delta \quad (3.13)$$

$$T = 0 \quad \text{at} \quad z = h(t)/\delta \quad (3.14)$$

$$T = 1 \quad \text{at} \quad z = 0, \quad (3.15)$$

and initial conditions;

$$T(z,0) = 0 \quad (3.16)$$

$$\phi(z,0) = 0 \quad (3.17)$$

$$h(0) = 0, \quad (3.18)$$

where $h(t)$ is the location of the partial melt boundary at time t . This boundary is determined by the equation:

$$\kappa \partial T / \partial z = \phi_0 L \rho dh / dt \quad (3.19)$$

Equations (3.7) through (3.11) with conditions (3.12) through (3.19) define, mathematically, the problem of melt migration in a growing layer of a binary partial melt, heated from below, in which thermodynamic equilibrium is maintained. The solutions to these equations would represent the zeroth order state, perturbations of which could lead to instabilities within the melt itself. Solutions to even the zeroth order state can be solved only numerically, except in the most trivial cases. Considerable insight into the importance of magma migration and matrix deformation on the zeroth order thermal regime can be gained, however, simply by examining the scale variables W_0 , δ , τ as a function of K_0 , η_f , by examining the dimensionless thermal conduction scale $a\tau/\delta^2$, and by comparing h to δ as a function of τ . The latter comparison depends upon the parameter S . For typical values $L = 10^5$ j/kg, $\Delta T = 100^\circ\text{C}$, $c_p = 10^3$ j/kg/ $^\circ\text{C}$, $\phi_0 \approx 0.1$, $S \approx 0.1$. Table 1 shows W_0 , δ , τ , and $a\tau/\delta^2$ for a range of values K_0 , η_f . Table 1 also

TABLE 1. Material Properties and Scale Variables as Functions of K_o and η_f .

(a) Material Properties and Constants:				
$\phi_o = 0.1$		$g = 10 \text{ m/s}^2$	s^2	
$d = 10^3$		$\zeta_s + 4/3 \eta_s = 10^{18} \text{ Pa-s}$		
$\Delta\rho = 500 \text{ kg/m}^3$		$a = 10^{-6} \text{ m}^2/\text{s}$		
(b) Scale Variables:				
η_f (Pa-s)	W_o (m/s)	δ (m)	τ (s)	$a\tau/\delta^2$
	$K_o = 10^{-10} \text{ m}^2$ ($\ell = 10^{-2} \text{ m}$)			
10^1	5×10^{-6}	3×10^3	6×10^7	6×10^{-6}
10^2	5×10^{-7}	10^3	2×10^8	2×10^{-4}
10^4	5×10^{-9}	10^2	2×10^9	0.2
10^8	5×10^{-13}	1	2×10^{11}	2×10^5
	$K_o = 10^{-12} \text{ m}^2$ ($\ell = 10^{-3} \text{ m}$)			
10^1	5×10^{-8}	3×10^2	6×10^8	6×10^{-3}
10^2	5×10^{-9}	10^2	2×10^9	0.2
10^4	5×10^{-11}	10	2×10^{10}	2×10^2
10^8	5×10^{-15}	10^{-1}	2×10^{12}	2×10^8

shows that, if $S \approx 0.1$, for high permeability and for low viscosity (i.e., basaltic) magmas, the conduction term in (3.9) is negligible, compared to the term involving compaction, whereas for highly viscous (i.e., granitic) magmas, compaction has virtually no effect on the temperature distribution in the partially molten layer. The zeroth order solution for "granitic" magmas is governed by thermal conduction. For intermediate magmas and high permeability or fluid magmas and low permeability, both conduction and compaction determine the thermal regime. Each of the extreme cases, termed compaction dominated and conduction dominated, respectively, is discussed below semi-quantitatively. By restricting the discussion to those cases where the relative importance of the various effects is clear, direct solution of the governing equations is not necessary. The recognition of the likely importance of instabilities within the melt during melt production and migration in a low viscosity basaltic melt on the dynamics of migration, the unimportance of such instabilities in high viscosity granitic systems, and the likely importance of shear in the dynamics of melt migration in the latter systems is all that is sought in this paper.

4. DISCUSSION

To fix the discussion, the ternary diagrams, the ternary represent low viscosity plagioclase phase diagrams, melts are considered great oversimplification granitic melts, but of the density behavior. The perturbation and 2, respectively. calculated for Fig. following discussion is the ternary phase diagram does not present serious physics of the process.

4.1. Compaction-dominated

Consider first the $\eta_f = 10^1 \text{ Pa-s}$, $K_o = 10^{-10} \text{ m}^2$ (3.9) reveals that compared to the compaction layer. The heat transfer will modify the motion of the melt. The conductive (Stefan) boundary that this can be neglected. The boundary is given, a

where the factor (0.1). Thus, in a unit of time ($t = 1$), $h \sim 35 \text{ m}$ or 10^{-2} . But if $t = 10^2$ but e^{-1} of the melt a constant initial pressure from that considered nevertheless their liquid-rich layer of A further complication partially molten layer unstable.

Consider the bulk composition. For bulk composition under equilibrium melt of a until all the melt is heated, diopside is melt. Thus, in a layer below, the density, temperature, and the melt occurs rapidly compaction however; and a relation

riables as Functions

τ (s)	$a\tau/\delta^2$
10^{18} Pa-s	
10^7	6×10^{-6}
10^8	2×10^{-4}
10^9	0.2
10^{11}	2×10^5
10^8	6×10^{-3}
10^9	0.2
10^{10}	2×10^2
10^{12}	2×10^8

ity and for low vis-
ion term in (3.9) is
compaction, whereas
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es in high viscosity
e of shear in the
ystems is all that is

4. DISCUSSION

To fix the discussion in terms of simple silicate phase diagrams, the ternary diopside-plagioclase phase diagram to represent low viscosity basic melts and the ternary quartz-plagioclase phase diagram to represent high viscosity silicic melts are considered. These phase diagrams are admittedly great oversimplifications of naturally occurring basaltic or granitic melts, but they do serve to show the important aspects of the density behavior as functions of composition and temperature. The pertinent phase diagrams are shown in Figs. 1 and 2, respectively. Note the effect of water on the densities calculated for Fig. 2 has been neglected. Because the following discussion is largely qualitative, the interpretation of the ternary phase diagrams in terms of two-component systems does not present serious obstacles to understanding the basic physics of the processes being considered.

4.1. Compaction-dominated systems

Consider first the compaction-dominated system given by $\eta_f = 10^1$ Pa-s, $K_0 = 10^{-10}$ m² in Table 1. An examination of (3.9) reveals that the thermal conduction term is negligible compared to the compaction term except in a very thin boundary layer. The heat transport that arises because of compaction will modify the motion of the phase boundary from the ordinary conductive (Stefan) case, but suppose for the sake of argument that this can be neglected. Then the motion of the phase boundary is given, approximately, by Carslaw and Jaeger [20]:

$$h(t) \approx 2(a\tau t/0.2)^{1/2}, \quad (4.1.1)$$

where the factor (0.2) comes from boundary condition (3.9). Thus, in a unit of time equal to the compaction time τ (i.e., $t = 1$), $h \sim 35$ m or $h/\delta \approx 10^{-2}$. Even if $t = 10$, $h/\delta = 3 \times 10^{-2}$. But if $t = 10$, Richter and McKenzie [8] show that all but e^{-1} of the melt is expelled from the porous layer, assuming a constant initial porosity. Their model is somewhat different from that considered here in that h is fixed a priori, but nevertheless their results suggest that for the current model a liquid-rich layer of considerable thickness ought to develop. A further complication may arise, however; the melt in the partially molten layer may be convectively or gravitationally unstable.

Consider the bulk compositions labelled *a* and *b* in Fig. 1. For bulk composition *a*, the composition of melts first formed, under equilibrium melting, lie along the cotectic to the left of *a* until all the plagioclase is melted. With additional heating, diopside is the only residual crystalline phase in the melt. Thus, in a layer of bulk composition *a*, heated from below, the density of the melt increases with increasing temperature, and the melt is gravitationally stable. Compaction occurs rapidly compared to the rate of growth of the layer, however; and a relatively liquid-rich melt layer is likely to

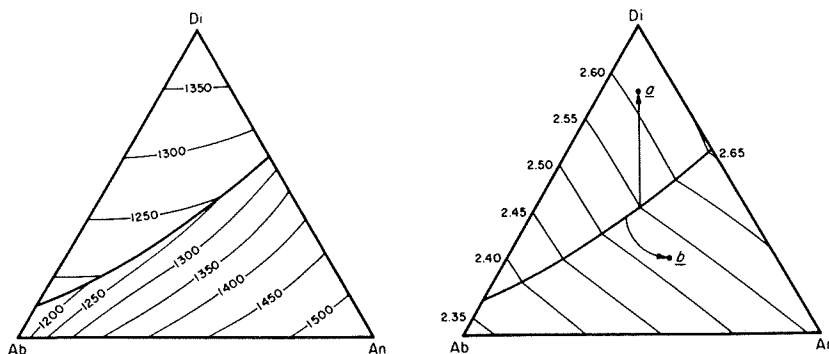


FIGURE 1. Phase diagram for Di-An-Ab. The temperature contours are plotted according to Weill *et al.* [17]. Liquid densities are calculated following Mo *et al.* [18]. *a* and *b* refer to assumed bulk compositions as discussed in text. The arrows show the liquid composition paths for the assumed bulk compositions.

be formed. If this layer forms by simple vertical flow, without mixing, it would have a destabilizing thermal gradient and a strongly stabilizing compositional gradient. The layer would become monotonically unstable to a finite amplitude perturbation when the thermal Rayleigh number for the layer exceeds the critical value of 10^3 [21]. Thus, instability in the melt-rich layer of thickness D would occur, approximately, when:

$$Ra = \frac{\rho \alpha g \beta D^4}{a n_f} \gtrsim 10^3 \quad (4.1.2)$$

With $\alpha = 3 \times 10^{-5}/^\circ\text{C}$, $\rho = 3 \times 10^3 \text{ kg/m}^3$, $\beta = 0.01^\circ\text{C/m}$, and other parameters given by Table 1, one obtains $D \sim 1 \text{ m}$.

Now consider bulk composition *b*, in the plagioclase field. As in case *a*, the composition of the first melts formed lie along the cotectic until all of the diopside is melted. The melt composition then moves off the cotectic and into the plagioclase + liquid field (see the arrow in Fig. 1). The melt having a composition on the cotectic is more dense than melt having a composition in the plagioclase + liquid field. Thus, for a layer of bulk composition *b* that undergoes equilibrium melting and is heated from below so that the temperature at the base is great enough for all the Di to be consumed, the melt at the top of the layer is more dense than melt at the base of the layer. This situation is gravitationally unstable, and one would expect a Rayleigh-Taylor instability to develop.

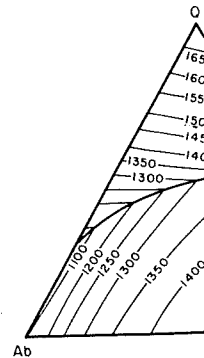
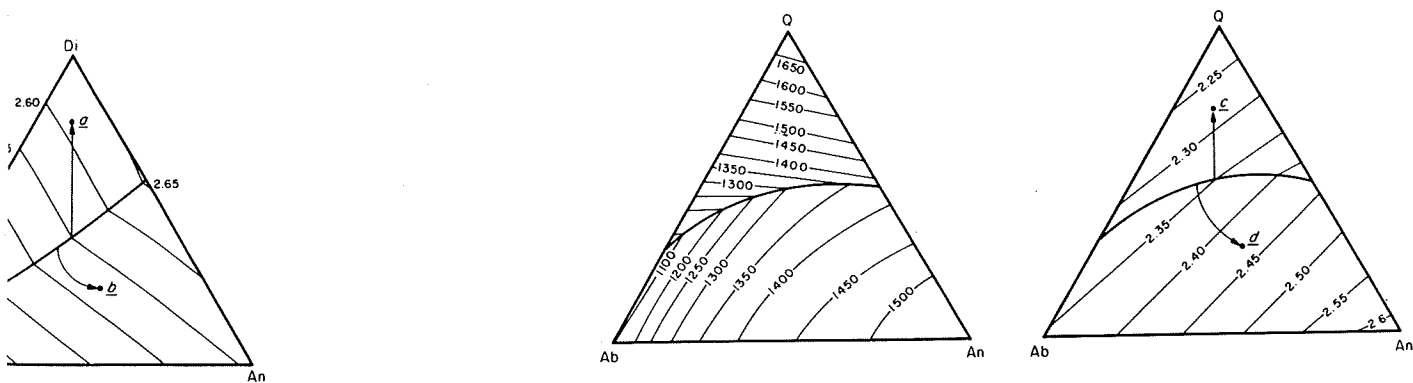


FIGURE 2. Phase diagram for Di-An-Ab, projected onto the $Di-An$ plane, excluding water. *c* and *d* refer to bulk compositions. The arrows show the liquid composition paths.

Admittedly, the Taylor instability in a melt are difficult to analyze. Compaction is rapid in a fluid rich, gravitationally unstable layer rather than in a solid. Such a layer would be unstable to Rayleigh-Taylor instability on a small scale. In the simple case [22] for two layers of equal thickness and density contrast $\Delta\rho$,

where D is the total thickness of the layer, is set equal to the critical value $\eta_f = 10 \text{ Pa-s}$, then

These results, from an analysis of the stability of low viscosity melts at low temperatures below are likely to be sensitive to the calculation of the effective viscosity of partially molten layers. The results suggest that Rayleigh-Taylor instabilities are likely to develop in such thin layers.



temperature con-
[17]. Liquid
[18]. a and b
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FIGURE 2. Phase diagram for Q-An-Ab at one atmosphere pressure, projected onto the anhydrous plane [19]. Liquidus densities, excluding water, are calculated following Mo et al. [18]. c and d refer to bulk compositions discussed in text. The arrows show the liquid composition paths for the assumed bulk compositions.

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Admittedly, the details of the development of a Rayleigh-Taylor instability in a growing, compacting layer of partial melt are difficult to describe even qualitatively. Because compaction is rapid in low viscosity, partially molten systems, a fluid rich, gravitationally unstable layer might be expected to form rather rapidly. It is interesting to see how thick such a layer would need to be for the growth time for the Rayleigh-Taylor instability to exceed the compaction time scale. In the simple example shown by Turcotte and Schubert [22] for two layers of equal thickness and viscosity, with a density contrast $\Delta\rho$, the minimum growth time is:

$$\tau_{R-T} = \frac{26\eta_f}{\Delta\rho g D} \quad (4.1.3)$$

where D is the total thickness of the melt-rich layer. If τ_{R-T} is set equal to the compaction time $\tau = 6 \times 10^7$ s, $\Delta\rho = 0.005$, $\eta_f = 10$ Pa-s, then $D \approx 10^{-4}$ m.

These results, though based on an extremely oversimplified analysis of the stability problem, strongly suggest that low viscosity melts produced in a compacting layer heated from below are likely to be highly unstable. This result does not appear to be sensitive to the starting composition, and though the calculation has been based on the model of a growing partially molten layer, it is difficult to imagine that the instabilities suggested here would not occur in an ascending diapir. It is very noteworthy that the melt instabilities would occur in such thin layers (\sim meters), because it implies that on the

spatial scales involved in melt production in the mantle, presumably of the order of kilometers to a few tens of kilometers, the instabilities would be extremely vigorous.

At the risk of carrying this simplistic analysis from the boundaries of reason to the realm of pure fantasy, three more possible implications of these results are suggested. First, the occurrence of instabilities in the melt changes the melt segregation problem from a one-dimensional one to a two- or three-dimensional one. One-dimensional models may offer very little insight into the dynamics of melt segregation in the mantle. The two- or three-dimensional nature of the motions in the melt would give rise to lateral changes in the motion of the melt front and locally enhance melt segregation at some places along the melt front. That is, diapirs may form on the melt boundary. Secondly, the melt would be expected to undergo vigorous mixing in the production zone. Major and trace element distributions in the segregated melt may represent compositions that have been averaged over a considerable range of pressure and temperature and not represent a melt batch having a simple eutectic composition. Lastly, because partial melt is continually generated at the top of the growing layer, even as an unstable, melt-rich layer is forming in the interior of the layer, one can see how zones of different melt fraction can be formed. Such a distribution of melt fractions may give rise to magmons [10].

4.2. Conduction-dominated systems

Now consider, for example, the case $\eta_f = 10^8$ Pa-s, $K_0 = 10^{-12}$ m² in Table 1. This system corresponds to a fine-grained granite. Clearly, the conduction term in the heat transfer equation (3.9) is much more important than the compaction term. Equation (4.1.1) is an excellent approximation to the growth of the layer. In a unit of compaction time, $t = 1$, for this system, $h_c \approx 6 \times 10^3$ m. Melt migration is negligible compared to the growth of the partially molten layer. The question of melt stability can be addressed as in the preceding section.

Consider as examples the bulk compositions c and d in Fig. 2. The analysis of the density structure in the partial melt is analogous to that in the preceding section. The principal difference, however, is that quartz replaces diopside as the third component of the phase diagram. Because quartz is less dense than plagioclase, whereas diopside is more dense, the density structure is reversed. The partial melt with bulk composition c (i.e., quartz + liquid field), shows a density inversion, whereas a partial melt with bulk composition d is stably stratified. In both of these cases, however, melt instability does not play an important role. Suppose one assumes that a melt-rich layer, equal to the compaction thickness, developed in a unit of compaction time in a liquid with bulk composition c . The growth time for the Rayleigh-Taylor instability in this layer, with the same parameters as used previously in (4.1.3), except $\eta_f = 10^8$ Pa-s and $D = \delta$, gives:

which is somewhat smaller than the Rayleigh number for this system (using the same parameters as used in the preceding section). Again, even if the instability would occur, the layer dimensions are so small that significant effects would be confined to the layer.

One is then led to conclude that melt segregation and compaction may play a major role in the dynamics of this system. This may be found in the Mt. Arabia migmatite is an example of partial melt and Covert [23], melt films; and with 25% melt, the melt coalescence is a result of shear and segregation in granitic fraction (permeability compaction).

From Table 1, $t_c \approx 2 \times 10^{11}$ s. The system is dominated by thermal conduction which is roughly equal to the time resulting from coalescence is of the order of 2×10^{11} s, or 10^3 m, which is a reasonable approximation, the granitic melts can be modeled by a one-dimensional compaction model that the general case present there is for basalt melt.

5. CONCLUSIONS

The dynamics of melt segregation in geology, and applied to the case when one places a melt in a one-dimensional compaction model of magnitude order of magnitude of low viscosity melts of low vis-

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$$\tau_{R-T} \approx 5 \times 10^{11} \text{ s}, \quad (4.2.1)$$

which is somewhat less than the compaction time. But an instability that occurred on a scale of 10^{-1} m could not be expected to be observed. The monotonic, finite-amplitude instability for this system (i.e., bulk composition \bar{d}), with the same parameters as used previously in (4.1.2), gives $D \sim 50$ m. The Rayleigh number is only 0.2 percent of the critical value. Again, even if the compacting layer were unstable, the instability would occur on spatial scales that are much less than the layer dimension. Such instabilities could not have a significant effect on the dynamics of the partially molten layer.

One is then left with the question as to how magma migration and melt segregation occurs in granite systems, because matrix compaction and melt instability appear to play such a small role in the dynamics of the partially molten system. An answer to this may be found, possibly, from the detailed field studies of the Mt. Arabia migmatite in Georgia [23]. The Mt. Arabia migmatite is an anatectic partial melt in which progressive degrees of partial melting may be observed. According to Size and Covert [23], shear structures are the sites of incipient melt films; and when the melt fraction reaches approximately 25%, the melt coalesces into larger braids and arteries as a result of shear and matrix deformation. Braids on the order of meters in width are observed. The critical features in melt segregation in granites may be a combination of a large melt fraction (permeability) and shear deformation as well as simple compaction.

From Table 1, one sees that if $K_0 = 10^{-10} \text{ m}^2$, $\delta = 1$ m, $\tau = 2 \times 10^{11}$ s. The factor $\alpha\tau/\delta^2 = 2 \times 10^5$, so the system is still dominated by thermal conduction. The compaction length scale, which is roughly equivalent to the length scale of melt migration resulting from shear, is roughly the scale at which melt coalescence is observed in the Mt. Arabia migmatite. In a time of 2×10^{11} s, equation (4.1.1) gives a layer thickness of $\sim 2 \times 10^3$ m, which is not unreasonable. It thus appears to a first approximation, that observed features of melt segregation in granitic melts can be explained on the basis of the two-phase flow model of McKenzie [7], though not on the basis of one-dimensional compaction alone. Nevertheless, it is interesting that the general theory has some credence in field data. At present there is no field verification of the McKenzie model for basalt melt segregation.

5. CONCLUSIONS

The dynamics of melt segregation is extremely important in geology, and appears to be exceptionally complex, particularly when one places thermodynamic constraints on the system. One-dimensional compaction models are useful for estimating the order of magnitude of the effect of matrix deformation. In melts of low viscosity, thermal conduction is unimportant, and

melt dynamics is controlled by a combination of matrix deformation and convective or Rayleigh-Taylor instability within the melt itself. These combined effects will give rise to two- or three-dimensional processes that may tend to focus the locales of melt migration. These processes have not yet been treated quantitatively in any fashion. One would suspect that the chemical signature of the source zone is considerably smeared as a result of melt instability in the production zone. It would be difficult to unravel such factors as degree of partial melting and other source characteristics in any more than an average sense.

On the other hand, in melts with high viscosity, thermal conduction dominates the thermal regime. Compaction and shear deformation act together to segregate the melt from the matrix. The melt is stable with respect to convection until large quantities of liquid have coalesced.

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APPENDIX

List of Symbols

- a - thermal diffusivity of solid-melt mixture ($a = \kappa/\rho c_p$)
 b - a constant relating mass fraction of light component to temperature assuming thermodynamic equilibrium
 c_p - specific heat at constant pressure
 d - a constant
 D - thickness required for melt layer to be unstable
 g - acceleration due to gravity
 h - location of the melt boundary
 K - permeability
 ℓ - grain size
 L - latent heat
 P - pressure
 Ra - Rayleigh number
 S - Stefan number $L\phi/\Delta T c_p$
 T - temperature
 T^* - temperature at base of layer
 T_e - temperature at top of layer (the eutectic temperature)
 u_i - velocity components in melt ($i = 1,2,3$)
 U_i - velocity components in matrix ($i = 1,2,3$)
 w - vertical velocity of melt
 W - vertical velocity of matrix

Greek Symbols:

- α - thermal expansion coefficient
 β - thermal gradient
 Γ_1, Γ_2 - rates at which components 1,2 are transferred from melt to matrix
 δ - compaction length (a scaling length)
 ζ - bulk viscosity
 η - shear viscosity
 κ - thermal conductivity of melt-matrix mixture
 ρ - density
 τ - compaction time (a scaling time)
 τ_{R-T} - growth time for Rayleigh-Taylor instability
 ϕ - porosity (volume fraction of melt)
 x - mass fraction of light component of melt

Subscripts:

- f - fluid
 o - reference value
 s - solid

Superscript:

- ' - dimensionless parameter

MELT MIGRATION IN

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ABSTRACT

1. INTRODUCTION
 2. A MACROSCOPIC
 3. APPLICATION T
 4. COMPACTION AN
 5. WAVES IN 2-D
 6. THE GEOPHYSIC
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