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MODERN METHODS OF IGNEOUS PETROLOGY:
Understanding Magmatic Processes

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PRINCIPLES OF THERMODYNAMIC MODELING OF IGNEOUS PROCESSES	*J. NICHOLLS*
THERMODYNAMIC PROPERTIES OF SILICATE LIQUIDS WITH EMPHASIS ON DENSITY, THERMAL EXPANSION AND COMPRESSIBILITY	*R.L. LANGE & I.S.E. CARMICHAEL*
SIMULATION OF IGNEOUS DIFFERENTIATION PROCESSES	*R.L. NIELSEN*
THE MATHEMATICS OF FLUID FLOW AND A SIMPLE APPLICATION TO PROBLEMS OF MAGMA TRANSPORT	*J. NICHOLLS*
PHYSICAL PROCESSES IN THE EVOLUTION OF MAGMAS	*S. TAIT & C. JAUPART*
MAGMA MIXING PROCESSES: INSIGHTS AND CONSTRAINTS FROM THERMODYNAMIC CALCULATIONS	*J.K. RUSSELL*
CONTROLS ON OXIDATION-REDUCTION RELATIONS IN MAGMAS	*I.S.E. CARMICHAEL & M.S. GHIORSO*
DYNAMICS OF ERUPTIVE PHENOMENA	*C. JAUPART & S. TAIT*
MELT FRACTION DIAGRAMS: THE LINK BETWEEN CHEMICAL AND TRANSPORT MODELS	*G. BERGANTZ*
TEXTURAL CONSTRAINTS ON THE KINETICS OF CRYSTALLIZATION OF IGNEOUS ROCKS	*K.V. CASHMAN*

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CHAPTER 9

G.W. BERGANTZ

MELT FRACTION DIAGRAMS: THE LINK BETWEEN CHEMICAL AND TRANSPORT MODELS

INTRODUCTION

The direct application of transport models to the long standing issues in petrology has often been a frustrating exercise. Rarely can one confidently identify detailed physical processes that drive petrologic diversity. Part of the difficulty originates in the construction of physical and chemical models; they often appear as fundamentally different enterprises. Chemical petrology has traditionally sought to characterize equilibrium states and to advance the formalisms of physical chemistry, the independent variables being taken from the usual thermodynamic ensemble: T, P, etc. Physical petrology differs in that the independent variables are space and time, and the effort is directed at identifying the structures and fluxes that follow from nonequilibrium states. It seems timely then to consider elements that are common to both chemical and physical descriptions of petrologic processes. A unification of this kind is guaranteed to succeed eventually as Nature does not seem to suffer the distinction.

The situation is particularily acute in the current generation of petrogenetic models. A quick review of the abstracts from the IAVCEI meeting on Continental Magmatism held in Santa Fe in 1989, reveals that petrologic diversity was attributed to magma mixing, crystal fractionation and partial melting in roughly equal proportions. Application of isotopic data has permitted estimation of the relative contributions of crustal and mantle "components" (Ruiz et al., 1988). When taken together in petrogenetic modeling, these processes are usually iterated until some degree of internal consistency is achieved. While all these assertions may well be true and data are available for a large number and variety of igneous suites, the issues regarding the time and length scales with which geochemical variations occur are not well constrained (Marsh, 1989a; Miller et al., 1988). The few studies where the temporal evolution is well constrained, e.g., Reagan et al. (1987) reveal markedly different time scales as a function of composition. Although many of the fluid models purported to mimic magma chambers have yielded a substantial literature on the behavior of pure fluids and salt solutions, less effort has been directed to the notion of geological constraints vis-à-vis geologic observables or the generation of testable hypothesis. This is testimony to the long absence of a detailed treatment of the most important element in magmatic systems: solidification.

Magmas are unlike other geophysical fluids, such as the oceans and atmospheres, in that their physicochemical history is dominated by phase changes. It is apparent from both laboratory and numerical experiments that many of the traditional formalisms of fluid dynamics can not be taken off the shelf and applied to magmatic systems, despite their convenience and the affection with which they are held (one notable exception being eruption processes and products). The self organization in crystallizing, viscous fluids is not a simple extension of that predicted by theory and experiment in constant property, single component fluids. The physical meaning of dimensionless numbers appearing in the governing equations for solidifying fluids must be interpreted and applied cautiously, a point well made by Bennon and Incropera (1987b) and Oldenburg and Spera (1990). Crystallization partitions the magma body into a region at the margins of solid and adjacent crystal-liquid "mush", and a fluid or "slurry" interior where crystallization occurs in an expanse of melt. The general physicochemical theory has been elucidated in the works of

Hills, Roberts and Loper (1978; 1983; 1988a,b,c; 1987), has been discussed in the geological context by Marsh (1988b; 1989b) and Morse (1988) and has been the subject of state-of-the-art numerical models originating in the engineering community (Beckermann and Viskanta, 1988a; Bennon and Incropera, 1987b; Thompson and Szekeley, 1989).

Our point of departure is to consider the generic magma system shown in Figure 1. A body of magma is cooling from the margin inwards and some time after intrusion we expect the system to partition itself into three regions: a region of solid which includes the country rock, an adjacent region of crystals and liquid (mush) where the melt fraction ranges from 0% to the that fraction at which the framework can no longer be self supporting, and a region of fluid magma (slurry). Depending on the phase diagram, initial geotherm, lithology of the country rock and thermophysical and transport properties, the regions of solid, mush and melt will distribute themselves according to the extant heat transfer processes. The primary question with respect to the schematic shown in Figure 1 is: what determines the partitioning of the crystallization processes between the mushy region and those in the interior? Both are potentially important in petrogenetic processes

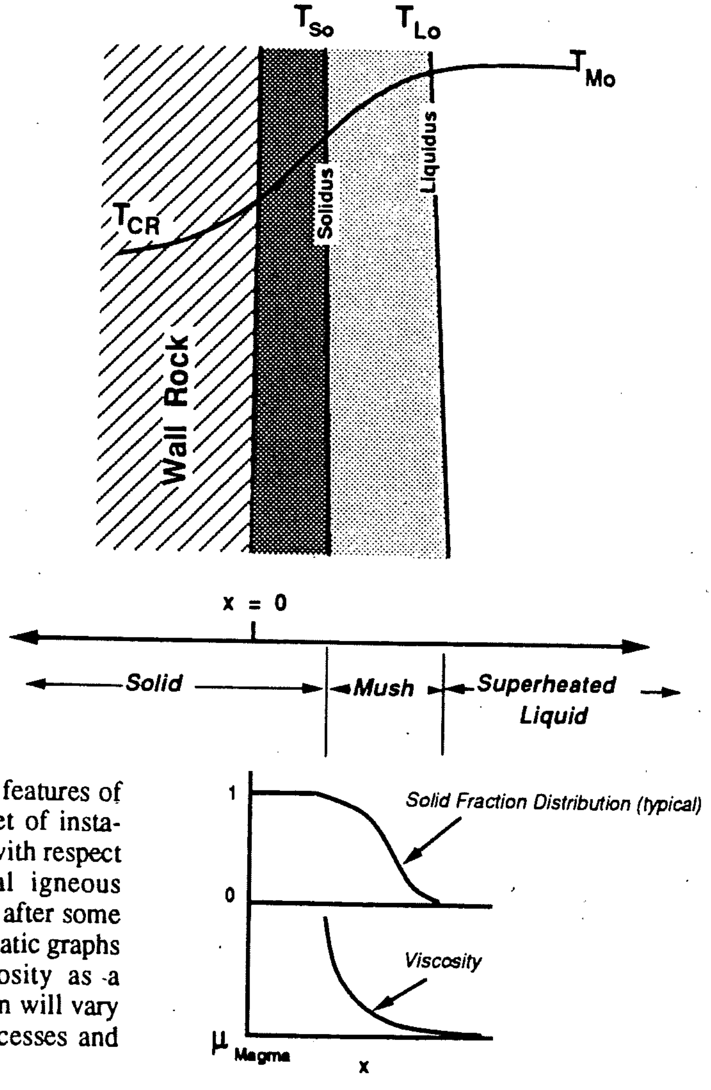

Figure 1. Coordinate system and general features of solidification problem. Prior to the onset of instability, the direction of growth is arbitrary with respect to gravity. The origin is the original igneous contact. The schematic shows the system after some time has passed; also shown are two schematic graphs of solid fraction distribution and viscosity as a function of distance. the actual distribution will vary depending on available heat transfer processes and thermo-physical properties.

(e.g., Langmuir, 1989; McBirney et al., 1985; Morse, 1988). In the mushy region crystal-liquid separation will depend on porous media flow, in the slurry crystal settling may occur. There is no question that substantial crystallization can occur in a distributed fashion in magma chambers as evidenced by numerous lava and ash flows with 10% to 50 % crystals. Much of the crystallization can also occur at the margins as documented at the Hawaiian lava lakes. Thus, when invoking crystal liquid separation, whether by solutol convection, crystal settling of some kind, magma mixing or partial melting, one needs to know the spatial and temporal distribution of mush and slurry (Marsh, 1988a). As discussed below, this also directly influences the rheology and hence the segregation mechanisms. Identifying the physical and chemical parameters that govern the partitioning of magmatic systems is one of the most geologically relevant challenges facing magma physics.

It is not difficult to appreciate that a treatment of transport phenomena should begin with as complete a solidification model as can be assembled. In this contribution we will consider one of the central components that links the thermodynamic treatments of magmatism with the attendant physical processes. This component is the melt fraction distribution, usually reported as a function of temperature: $f_m(T)$.

MELT FRACTION DISTRIBUTIONS

A general energy conservation expression

The extremely detailed structure of a crystal-liquid mush precludes direct, quantitative description. An alternative macroscopic approach is to invoke mixture theory which describes, in an formally average sense, the local state of the system. Mixture theory has provided a very serviceable description of flow in porous media and has recently been adapted to the study of crystallization (Beckermann and Viskanta, 1988a,b; Bennon and Incropera, 1987a,b; Oldenburg and Spera, 1990). A portion of the discussion below follows directly from these works and the interested student of transport is directed there for a thorough discussion of the continuum treatment of mixture theory. The fundamental components of mixture theory are equations for the conservation of mass, energy, linear momentum and chemical species. These usually form a formidable array of coupled, nonlinear partial differential equations which precludes direct interpretation in terms of a single dimensionless number or length scale. We will focus on only one of the conservation expressions: the conservation of energy, for it is here that the influence of the physical chemistry on the transport is most readily apparent.

The expression for conservation of energy is typically cast directly in terms of temperature, however there are advantages in considering the more primitive thermodynamically dependent variable enthalpy. The advantages follow from the fact that the temperature can be written as a continuous function of enthalpy and hence the most formidable nonlinearities attendant with phase change problems can be recast in terms of the relationship between the enthalpy, h, and temperature (Cao et al., 1989; Hunter and Kuntler, 1989). In practice this means that enthalpy can be written as a continuous variable and in numerical models, cpu intensive front tracking techniques can be avoided. Notable treatments of the enthalpy method to solve solidification problems are those of Baxter (1962), Shamsundar and Sparrow (1975), and Voller (1985).

Consider a general expression for the conservation of energy in an incompressible fluid where there is no viscous dissipation or heat sources such as radioactivity decay. Local thermal equilibrium is assumed, the temperature of adjacent liquid and solid phases is assumed to be the same. In a fixed volume the conservation of specific enthalpy for an assemblage of n phases is

$$\frac{\partial}{\partial t}\left(\sum_{i=1}^{n} g_i \rho_i h_i\right) + \nabla \bullet \left(\vec{u}\sum_{i=1}^{n} g_i \rho_i h_i\right) = \nabla \bullet \left(\sum_{i=1}^{n} g_i k_i \nabla T\right), \tag{1}$$

where t is time, g_i is the volume fraction of phase i, ρ_i the density, h_i the specific enthalpy, k_i the conductivity, T the temperature, u the velocity, n the total number of phases, and the vector operators have their usual meaning. We will initially only consider systems where all the crystallization occurs at the boundaries and the porous crystal framework is infinitely rigid, hence the mush zone is delimited by the solidus and liquidus isotherms. Assuming constant heat capacity and that the difference between the enthalpy content in coexisting liquid and solid phases in the volume is equal to the latent heat, (1) can be written as:

$$\rho c^* \frac{\partial \theta}{\partial \tau} + \vec{U} \bullet \nabla \theta = \nabla \bullet \left(k^* \nabla \theta\right) + \frac{1}{\rho_l c_l (T_L - T_S)} \sum_{j=1}^{n-1} \left(\rho_l c_l - \rho_{sj} c_{sj}\right) \frac{dg_{sj}}{d\tau}, \tag{2}$$

where the subscript s indicates a solid phase, l the liquid phase and where

$$\rho c^* = \frac{g_l \rho_l c_l + \sum_{j=1}^{n-1} g_{sj} \rho_{sj} c_{sj}}{\rho_l c_l}, \qquad k^* = \frac{g_l k_l + \sum_{j=1}^{n-1} g_{sj} k_{sj}}{k_l}, \tag{3}$$

and

$$\tau = \frac{t\kappa_l}{L^{2}}, \qquad \vec{U} = \frac{\vec{u}\cancel{\kappa_l}\,L}{\cancel{L}\,\kappa_l}, \qquad \theta = \frac{T - T_S}{T_L - T_S}, \tag{4}$$

where κ_l is the liquid molecular thermal diffusivity, L the length scale in the direction of solidification, T_L the liquidus temperature, T_S the solidus temperature and where we invoke the saturation condition:

$$g_l + \sum_{j=1}^{n-1} g_{sj} = 1. \tag{5}$$

Note that temperature differences are normalized to the difference between the solidus and liquidus temperatures. This could be relaxed as Equation (2) can be written as a general equation for the entire expanse of solid-crystals-melt (Beckermann and Viskanta, 1988b; Bennon and Incropera, 1987a), however we wish to focus on the enthalpy changes associated specifically with phase change. When written in conjunction with ancillary equations for conservation of energy in solid and liquid only regions, Equation (2) is entirely consistent with mixture theory.

Equation (2) is difficult to apply in practice as it requires that the specific heats and latent heats be known for every phase as well as the instantaneous volume fraction of each phase as required by the latent heat term, the last one on the right hand side of (2). The large number of phases and complex exchange equilibria make it difficult to construct thermodynamically consistent forward models of phase volume fractions and system properties as a function of the intensive variables, notable exceptions being the models of

Ghiorso (1985) and Nekvasil (1988). In the absence of a complete thermodynamic formulation, phase equilibria experiments provide the only means by which to reliably obtain phase volume fractions. Unfortunately, this data is rarely reported and often only as volume percent melt and hence detailed knowledge of the volume changes in solid phases is usually unavailable. One expedient is to consider the system as an effective binary. This requires only that the volume melt fraction as a function of temperature be known. Thermophysical properties such as specific heat capacity and latent heat for the solid phases will need to be chosen to represent effective average values. While not entirely satisfying, such devices permit us to combine the numerical methods and laboratory data at hand.

If we chose to represent the system as an effective binary, Equation (2) will be written as:

$$\rho c^{*}\frac{\partial\theta}{\partial\tau}+\vec{U}\bullet\nabla\theta=\nabla\bullet\left(k^{*}\nabla\theta\right)-\frac{d\theta}{d\tau}\sum_{k=\cancel{0}\,1}^{i}\frac{\theta^{k-1}}{Ste_k}, \tag{7}$$

where the Stefan number, Ste_k, is defined as:

$$Ste_k=\frac{c_s\left(T_L-T_S\right)}{\Delta h_{l-s}ka_k}, \tag{8}$$

and the volume melt fraction has been written as a general polynomial in dimensionless temperature:

$$g_l=f_m(\theta)=\sum_{k=0}^{i}a_k\theta^k. \tag{9}$$

The Stefan number(s), (8), are actually a sequence of dimensionless coefficients providing a measure of the sensible heat to the latent heat content in the crystal-liquid region. They are the dimensionless group that establishes the proper similitude for comparison of numerical and experimental results against real systems. The Stefan number(s) given in (8) also include the influence of the slope of the solid fraction distribution curve. For a system with a linear (constant slope) melt fraction distribution, i equal to one, a single Stefan number is obtained; the additional terms in the sequence account for the higher order derivative terms associated with a nonlinear distribution; the form given in (8) explicitly accounts for the nonlinearity of the melt fraction distribution. The nonlinearity due to the latent heat term of (7) appears explicitly for i greater than one.

In metallurgical and engineering applications the melt fraction distribution is often assumed to be linear and the lever rule is used to describe the melt variation with temperature. Examples and discussions of the expressions for the melt fraction distribution have been given by Clyne and Kurtz (1981) and Hills and Roberts (1988a) and are based on a modified form of the Scheil equation. The melt fraction distribution for geological materials need not be linear, see Figure 2, and the utility of writing the melt fraction, $f_m(T)$, as a general nonlinear function is immediately apparent. The coefficients in (9), a_k, are obtained by curve fitting experimental data. An additional and important consideration is that the melt fraction during melting may not be a simple reversal of the solidification curves as the crystals may be zoned and present a changing compositional face as melting proceeds (Hills and Roberts, 1988b; Roberts and Loper, 1987). In addition, volatile constituents may exist metastably in the form of structurally bound water in the inosilicates and phyllosilicates and upon melting yield large jumps in melt percentage over small

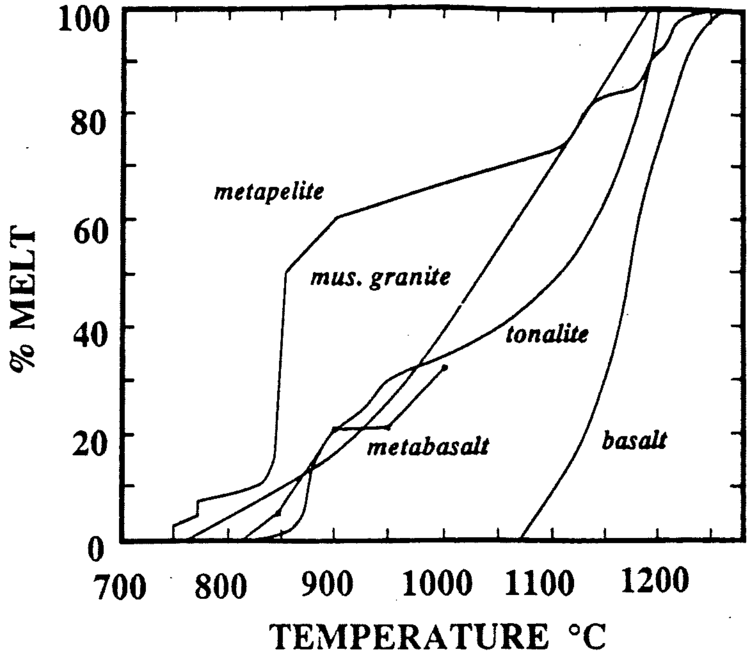

Figure 2. Melt fraction as a function of temperature for five rock types: basalt (Marsh, 1981), muscovite granite (Wyllie, 1977), tonalite (Rutter and Wyllie, 1988), metapelite (Vielzeuf and Holloway, 1988) and metabasalt (Beard and Lofgren, 1989). Patiño Douce and Johnston (1990) give a metapelite melting curve that is approximately linear between 800 to 1000°C.

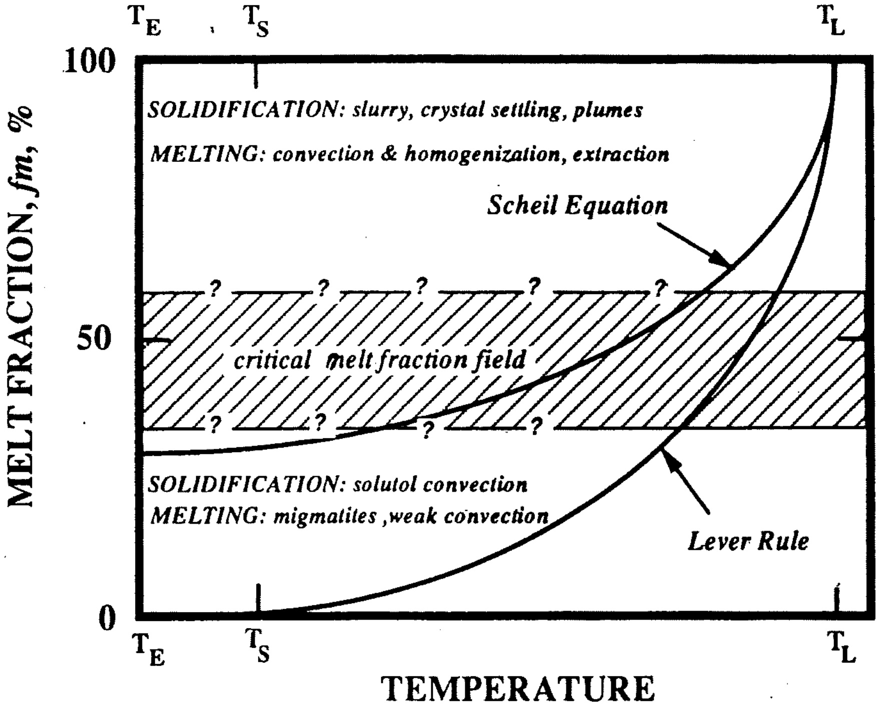

Figure 3. Schematic melt fraction as a function of temperature. Scheil equation and lever rule provide an envelope for natural systems as a function of crystal-liquid equilibrium. As shown, the critical melt fraction field is not truly a function of temperature and is drawn to represent the range of experimentally determined values.

temperature intervals as seen in Figure 2. A heterogeneous spectrum of crystal sizes will also influence the rate at which melting proceeds (Sheckler and Dinger, 1990). Finally, caution must be exercised when using the $f_m(T)$ obtained from experimental data, such as that in Figure 2, in transport models. These curves are constructed by interpolating between data points that represent discrete experiments where it is implicitly assumed that the kinetics of passing from one experimentally determined value of the solid fraction to the next along the $f_m(T)$ curve are negligible. One useful assumption is the fast-melting approximation of (Hills et al., 1983), which simply assumes that changes in temperature, no matter how small, yield an immediate and concomitant change in the local percent solid. Note that this does not imply that chemical equilibrium is maintained, e.g. diffusion in the solids may be neglected. This assumption seems to be applicable in the solidification portion of the coupled solidification/melting problem as nucleation is heterogeneous and under-coolings are demonstrably small (Cashman and Marsh, 1988). A discussion of the kinetics of reaction in the context of a general theory for the physiochemical evolution of a multicomponent system can be found in (Ghiorso, 1987; Hills and Roberts, 1988a).

Generic elements of melt fraction curves

In addition to playing a central role in the heat transfer, the melt fraction distribution largely determines the rheological partitioning and subsequent chemical evolution of the body. This partitioning will in turn delimit portions the phase diagram in which certain segregation processes may dominate, say crystal settling as opposed to solutol convection. The permeability and yield strength of the mush, and temperature and compositional differences of the mush and slurry, will also depend on the degree of chemical equilibrium between crystals and melt.

To appreciate some of the generic aspects of melt fraction distributions, consider Figure 3. Schematic melt fraction curves are shown and have been drawn as smooth curves for illustrative purposes. The abscissa is temperature with T_L being the liquidus, T_S the solidus and T_E the eutectic temperature. For a multicomponent system with solid solution the melt fraction distribution will depend on the degree of equilibrium maintained, or similarly whether crystals are removed. If equilibrium can be maintained, the lever rule provides the expression for melt fraction where it has been written for a binary with solid solution as (Bennon and Incropera, 1987a; Clyne and Kurtz, 1981):

$$f_m(T) = 1 - \frac{1}{1-k}\left(\frac{T_L - T}{T_F - T}\right) \tag{10}$$

where T_L is the liquidus temperature for a given composition, T_F the fusion temperature of the pure end member component and k is the partition coefficient (not to be confused with the use of k as the thermal conductivity in the preceding section). The partition coefficient is the ratio of the slope of the liquidus to solidus curves; if the liquidus and solidus lines are straight then k is a constant. In general it also will be a function of temperature. For perfect fractionation, the Scheil equation can be invoked to describe the melt fraction:

$$f_m(T) = \left(\frac{T_F - T}{T_F - T_L}\right)^{1/(k-1)}, \tag{11}$$

The nonequilibrium melt fraction will extend across a greater temperature range than the equilibrium case as the composition is constantly changing. This has been aptly demonstrated by comparing the percent crystallization curves for equilibrium and nonequilibrium crystallization of olivine tholeiite as given by Figs. 13 and 5 of Ghiorso (1985). The crystallization interval for the equilibrium case is about 150 °C, for the nonequilibrium case about 400 °C. The lever rule and Scheil equation provide an envelope for natural systems, the actual position depending on the components, the ability of diffusion in the solid to sustain equilibrium, and the rate of heat removal.

Spanning this envelope in Figure 3 is a field labeled critical melt fraction. The intent is not to portray this field explicitly as a function of temperature but rather just to identify that portion of the diagram consistent with existing experiments. This field delimits the region of mush from the slurry. Recall that the mush was defined to be the region of crystals and liquid where the melt fraction was low enough to yield an effectively rigid framework of crystals. The slurry is that portion where crystals exist in an expanse of melt. This rheological subdivision of the chamber can then be mapped directly onto the phase diagram, or melt fraction distribution curve in the absence of a phase diagram. The self partitioning of a multi-phase system was explored in the axiomatic works work of Brandeis and Marsh (1989; 1990) who introduced the concept of the convective liquidus. The convective liquidus is defined as the isotherm associated with the transition from a mush to a slurry. Thus, when identifying a temperature difference to scale thermal convection in the slurry, the temperature difference will be that between the intrusion temperature and the convective liquidus. One implication of this work is that the temperature differences available to drive convection in multicomponent systems are dynamic quantities and do not necessarily follow from initial temperature differences and geometry (again see discussions by Bennon and Incropera (1987b) and Oldenburg and Spera (1990)). A related concept originating in the metallurgical literature is the concept of the contiguity limit. Contiguity is a measure of solid-to-solid grain contact in partially molten systems (German, 1985; Miller et al., 1988; Ryerson et al., 1988) and the contiguity limit is the volume melt fraction at which a "rigid" aggregation of crystals takes on a suspension like behavior. In the geophysical literature this transition is called the critical melt fraction (Arzi, 1978; van der Molen and Paterson, 1979; Wickham, 1987).

Identifying the convective liquidus in real systems is difficult; there is little unequivocal geological data to indicate what melt fraction corresponds to the convective liquidus. In the experiments of Brandeis and Marsh, as in the Hawaiian lava lakes, the convective liquidus was approximately the intrusive temperature. Lava flows have a upper limit of crystallinity of ~50% (Marsh, 1981); granitic rock undergoing melting by the flow of basalt in the feeder dikes of the Columbia River basalt begins to disaggregate at about ~50% melt (W. Taubeneck, oral comm., also see Kitchen (1989)). Experiments by Jurewicz and Watson (1985; Miller et al., 1988) also suggest a critical melt fraction of ~50%. At "high" strain rates, $10^{-5}s^{-1}$, van der Molen and Paterson (1979) propose that the critical melt fraction is 30-35%. Experiments on partially molten granitic aggregates by Dell'Angelo and Tullis (1988) display a variety of deformation mechanisms from cataclasis to melt enhanced deformation as a function of melt fraction in the range of 0-15%. From the available examples, it is clear that the constitutive relations of a partial melt will depend on the strain rate, physical chemistry and grain size, making it difficult to generalize the rheology of partial melts.

Viscosity in magmatic liquids is primarily a function of temperature, crystal content and composition (McBirney and Murase, 1984; Ryerson et al., 1988; 1988; Spera et al., 1982). The viscosity of rhyolites can change up to seven orders of magnitude over approximately 700 °C, the viscosity of tholeiitic basalt by two orders of magnitude over 400 °C. It is important to note that in the physical modeling of the flow of interdendritic liquid the appropriate viscosity expression will be that of a crystal free fluid, although still temperature and compositionally dependent, and formulations such as those given by Shaw (1969; 1972) and Bottinga and Weill (1972) can be used. For convection in the slurry where intratelluric crystals will be present, the viscosity expressions of Ryerson et al. (1988) and Spera et al. (1988) should be employed. Strongly temperature dependent viscosity influences convection by partitioning the flow field into a region of cold, viscous boundary layers where the strain rates are an order of magnitude (typically more) less than that of the free fluid, thus creating an effectively smaller container where the temperature differences available to drive convection are those between the interior and the temperature where the viscosity increases by about an order of magnitude. Although the variable viscosity reorganizes the flow filed, the parameterization of the heat transfer can be done with the usual dimensionless groups where an effective viscosity is used in place of the true viscosity (Chu and Hickox, 1990). Variable viscosity strongly inhibits the efficiency of solutol convection as a means of melt segregation. This is simply and dramatically shown in the work of Tait and Jaupart (1989) and Chen and Thangram (1982). Tait and Jaupart point out that there are two time scales to be considered in extrapolating laboratory experiments to real systems, one for crystal growth and one for the convective instability. The net result is that solidification proceeds on essentially conductive time scales. Although some local compositional convection did occur in the experiments, it is not clear that this will occur in geological systems where the fluid density changes are not monotonic and viscosities relatively high. The net result is that fluid experiments with low and constant viscosity salt solutions may show a degree of self organization that is not consistent with real magmas.

The specification of the permeability in the magmatic mush is difficult; bear in mind that permeability K, like thermal and species diffusivity, is a tensor quantity and is described by nine coefficients. The permeability is an important property when estimating the efficiency of intercrystalline flow driven by the buoyancy changes associated with crystallization. One expression for steady fluid velocity in a porous media is given by Darcy's Law (Lowell, 1985):

$$\vec{u} = -\frac{K}{\mu}\left[\nabla P + \rho \vec{g}\right], \tag{12}$$

where u is the fluid velocity, K the permeability, μ dynamic viscosity, ρ density and g the acceleration of gravity. Note that one important parameter group is the ratio of the permeability to the dynamic viscosity. They vary in a sympathetic manner with regard to the efficiency of porous flow: when the permeability is low the viscosity will be high. The net effect being an inhibition of flow. Conversely, when the permeability is high the viscosity will be lower.

Permeability expressions are usually constructed as a function of porosity by invoking a simple geometrical model, such as a ball and stick or parallel plates, for the porous framework (Bear, 1972). It is commonly assumed that the permeability is locally homogeneous and isotropic: all the off diagonal terms are zero and the on diagonal terms have the same magnitude and so are functionally scalar quantities. One expression for permeability commonly employed is the Kozeny-Carman equation (Bennon and Incropera, 1987b):

$$K = K_o\left[\frac{f_m^{\,3}}{(1-f_m)^2}\right], \tag{13}$$

where the constant K_o is a function of the detailed structure of the crystal interfaces. Beckermann and Viskanta (1988b) use an expression similar to (12):

$$K = \frac{d_m{}^2 f_m{}^3}{175(1-f_m)^2}, \tag{14}$$

where d_m is the mean diameter of the crystals. An alternative form based on a microscopic flow between parallel plates is (Lowell, 1985):

$$K = \frac{d_m{}^2 f_m{}^2}{72\pi}, \tag{15}$$

Permeability expressions (13-15) are thought to be most appropriate for moderate melt fractions, f_m less than about 50%. In practice, determining the permeability is largely an empirical exercise.

It is not clear how to extrapolate from the existing body of laboratory work on melt distribution (Daines and Richter, 1988; Jurewicz and Watson, 1985; Waff and Bulau, 1982) to high melt fractions and to establish an expression for permeability as a function of temperature and hence percent melt. Although the laboratory experiments provide an expression for the melt fraction as a function of temperature, it is less clear how an interconnected network forms. The experiments of Daines and Richter (1988) show a remarkable degree of interconnectivity: almost all melt pockets were connected at melt fraction as low as 2% in a powered mixture of olivine and basalt powder. Thus, perhaps it is a safe assumption that the porosity can be used directly in the permeability expressions given above.

The melt fraction partitions the phase diagram by virtue of rheological constraints and by controlling the efficiency of solutol convection. One feature of Figure 3 is that the temperature differences available to drive thermal convection and efficient crystal liquid segregation will be, at most, the difference between the intrusion temperature and the temperature corresponding to the intersection of the melt fraction curve and the critical melt fraction field. The efficiency of fractionation driven by the density changes attendant with crystallization in the mush will depend critically on the ratio of the permeability to the dynamic viscosity.

GEOLOGICAL IMPLICATIONS

The MASH hypothesis and a baseline physical state

One interesting new paradigm for convergent margin magmatism is the MASH, melting-assimilaltion-storage-homogenization hypothesis of Hildreth and Moorbath (1988) that was developed to explain arc magmatism in the Andes. This carefully documented work suggests that much of the base-level chemical signature of arc magmas is generated in the deep crust by repeated interaction, i.e. underplating and penetrative intrusion of mantle basalts with deep crust. A variety of compositions can be generated by melting rocks with cratonic affinities, or perhaps more importantly, by remelting previous underplated material with subsequent homogenization. This process ultimately yields a "steady-state" region at or above the basalt solidus with a concomitant reduction in the ductility contrast. These highly ductile interactions have been documented in the field (DeBari et al., 1987). Thus, the usual viscosity barriers that inhibit differentiation (Marsh, 1988a) are removed and a wide variety of physicochemical interactions are possible. A similar hypothesis has been developed in island arcs to explain the origin of andesite (Takahashi, 1986).

There are two corollaries that follow from the MASH hypothesis: first, there is a base-level physical state that could provide a conceptual framework for the physical history as the base-level chemical signature does for the chemical history. Secondly, that magmatism is explicitly a crustal scale phenomena and the notion and quantification of path dependence should be addressed. The good news is that this clears the air for paradigms whose content may involve as much structural and metamorphic as igneous petrology. The bad news is it requires field and lab work on a scale difficult to manage and fund in a geological environment where no one is sure what is signal and what is noise. The length scales over which heterogeneity are manifested are highly variable depending on the quantity being measured, e.g. isotopes, major elements, etc., as discussed by (Hill et al., 1985; Miller et al., 1988). In the hunt for generic elements in a magmatic system, establishing a possible parent magma or isotopic provenance is only one of a number of elements that will ultimately comprise a sufficient "explanation".

The baseline physical state will be determined by the competing processes of penetrative intrusion of basaltic magmas, partial melting, ascent and segregation. The melting of lower to mid crustal rocks to generate petrologic diversity has been invoked: in Mexico, Mahood and Halliday (1988) find that the stable and unstable isotopic data are consistent with a origin of the rhyolites from melting of lower crustal rocks; in the central Cascades, Smith and Leeman (1987) suggest that mantle derived magmas induced melting at crustal levels to generate dacitic magmas. Experimental work by Beard and Lofgren (1989) indicates an origin for tonalites by melting of metamorphosed mafic rocks. Experiments by Conrad et al. (1988) show that the silicic magmas of the Taupo Volcanic zone could have originated by partial melting of a metaluminous source. The major and trace element and isotopic character of some plutonic rocks is also consistent with an origin by partial melting of mafic to intermediate deep crust, and subsequent mixing with partial melts of more cratonic affinities (Chappell and Stephens, 1988; Fleck and Criss, 1985; Gromet and Silver, 1987; Hawkesworth et al., 1982; Hill et al., 1988; Hill et al., 1986; Saleeby et al., 1987; Whitney, 1988).

The large volumes and extended duration of silicic magmatism in extensional regions appears to require a continuous source of more primitive and hotter magma to generate and sustain long lived magmatic centers, e.g. Halliday et al. (1989). This has been discussed in some length by Barton (1989), Hildreth (1981) and Whitney (1988) and proposed more recently in Nevada and Utah by Gans et al. (1989), Best et al. (1989) and Warren et al. (1989). One particularily interesting example is the relationship between the Timber Mountain Tuff and the Black Mountain plutonic complex which may provide an exceptional opportunity to explore the relationship between mafic plutonism and silicic ignimbrites (Vogel and Cambray, 1990). Glazner and Ussler (1988) argue that these underplated basalts can be geophysically imaged and are responsible for midcrustal increases in the P wave velocity. It has also been proposed that this underplating and penetrative intrusion of basaltic magma at upper crustal magma chambers is probably the source of enclaves in plutonic rocks and mingled textures in volcanic rocks (Bacon, 1986; Grove et al., 1988), and has also been proposed to be the cause of eruptions (Eichelberger, 1980; Warren et al., 1989).

The generic element in all the studies given above is the transport of heat and mass by mafic magmas, and the role of crustal melting; including young mafic crust that is isotopically indistinguishable from mantle, see discussion in Davidson et al. (1987). This does not preclude closed system crystal fractionation as an important process. The distinction between crystal fractionation and partial melting may well be irrelevant, or at least geochemically indistinguishable, in a region that is maintained at or near the basalt solidus and subject to re-intrusion. This could yield a magmatic complex subject to some of the kind of melting and extraction phenomena described by Naslund (1986), Beard (1990) and Fountain et al. (1989).

The extent of partial melting and subsequent convective homogenization will depend on a variety of factors: the initial temperature contrast between the basalt and the surroundings, the melt fraction distribution, and the thermophysical properties of the materials (Bergantz, 1989b; Fountain et al., 1989). The melt fraction is critical as it determines both the efficiency of porous media convection and compaction in moving the melt away from the body. For a single episode of underplating, the maximum melt fraction at the contact is plotted as a function of initial temperature in Figure 4; the calculations are discussed in Bergantz (1989b). The contiguity limit was taken to be ~50% and is shown as a dashed horizontal line. The important point is that the host lithology exerts a strong influence on the subsequent transport by virtue of the melt fraction distribution. In these calculations it was assumed that the two phase region was infinitely rigid and hence no compaction could occur. Fountain et al. (1989) incorporate a compaction model with underplating and demonstrate that the coupled two phase flow can yield significant amounts of magma that will collect into a chamber. The upward progress of this seems limited to less than about the half width of the mafic intrusion.

Both the models of Bergantz (1989b) and Fountain et al. (1989) assumed that the heat transfer can be adequately characterized by conduction. Huppert and Sparks (1988) have explored the interaction between basalt and crust with laboratory experiments that demonstrate the variety of fluid instabilities that can occur during underplating with vigorous convection. These models find their greatest application in systems with substantial amounts of superheat as their "basalt" was superheated and had a lower melting temperature than the countryrock. Experiments done by Bergantz (1989a), in which the relationship of melting ranges between the model "basalt" and "countryrock" was maintained and the "basalt" was intruded at the liquidus did not demonstrate vigorous convection. These differing results indicate that geological systems can have diverse responses to underplating depending on the degree of superheat, supercooling, melt fraction distribution and viscosity relations.

If the magma is superheated, the convective liquidus is very near the solidus or the system is open, such as flow in a dike, the progress of solidification will undergo some interesting time dependent behavior. This has been explored by in a number of papers in the engineering literature, e.g. Cheung and Epstein (1984) and subsequently in the geological case by Huppert and Sparks (1989). These formalisms have been extended by Bergantz (1990) for systems with a temperature dependent solid fraction distribution. Initial cooling at the margins upon intrusion or dike flow will yield a region of mush whose thickness will reach a maximum and then retreat in the face of a continued thermal flux supplied by convection. If the enthalpy content of the chamber is infinite then the early formed region of mush and solid will retreat, ultimately yielding a contact that has the same temperature of the flowing melt, providing the thermal conductivities are the same. If the enthalpy content is finite, an oscillation can occur: solidification and mush growth at early times, followed by retreat of the mush as convection proceeds and the mush may not retreat all the way back to the contact as the vigor of convection itself is decreasing. As convection dies the mush thickness grows again. This is shown in Figure 5 (see Bergantz

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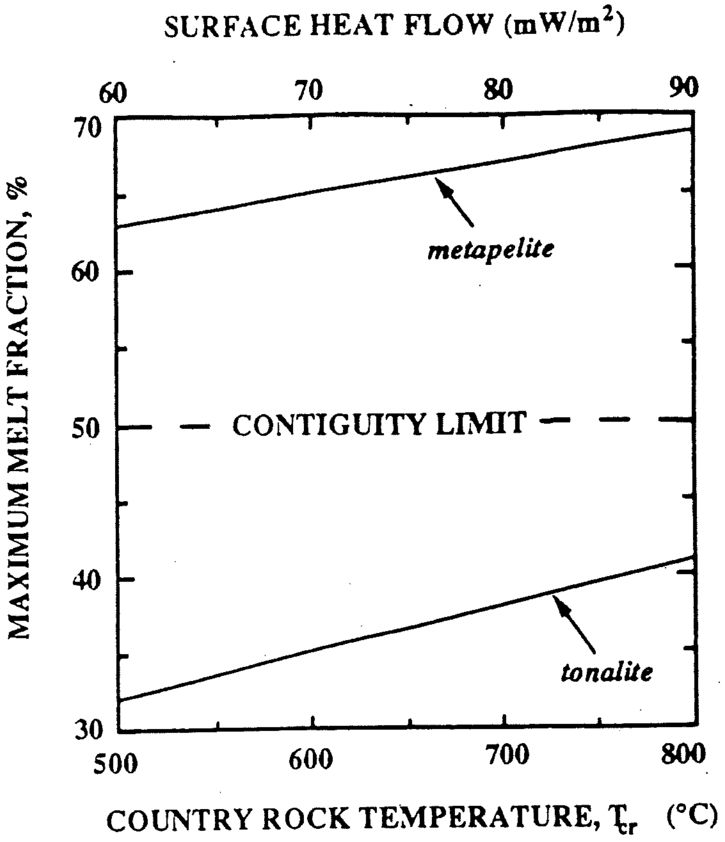

Figure 4. Maximum melt fraction as a function of the ambient temperature in the country rock. This maximum will occur near the contact and is not the average melt fraction of the partially molten region. Typical surface heat flow is plotted along the upper axis, basalt temperature at time of underplating is 1250°C.

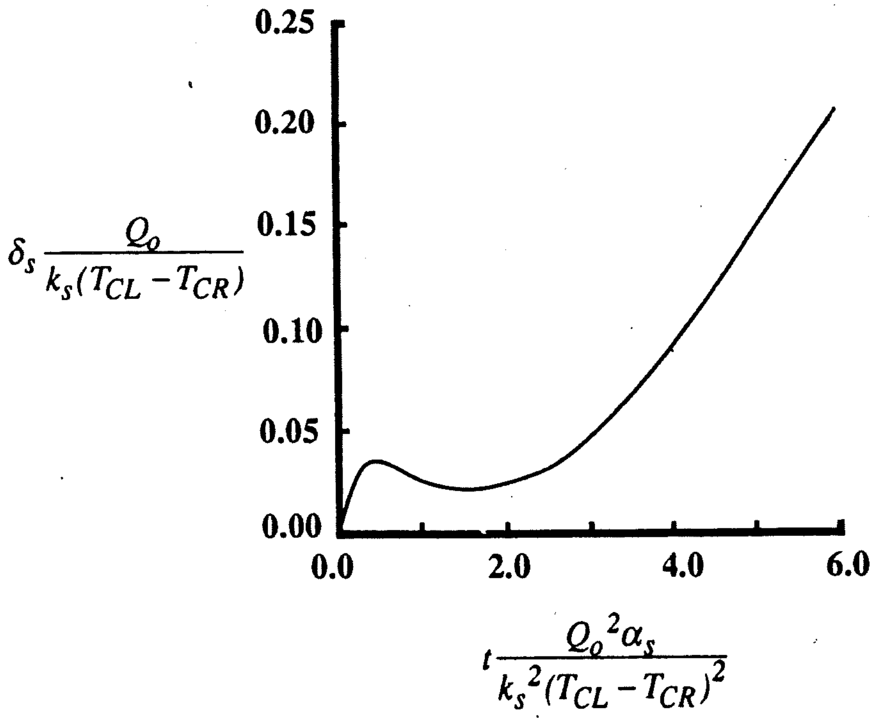

Figure 5. Dimensionless position of solidus isotherm relative to contact as a function of dimensionless time for a closed, convecting multicomponent system where melting is also occurring at the margins (Bergantz, 1990). Parameters: Q_o is convective heat flux at time equal zero, a the thermal diffusivity, k conductivity, T_{CL} convective liquidus temperature, T_{CR} country rock temperature in the far field, t time, d position of the solidus.

(1990) for details of the calculations) which is a plot of the dimensionless thickness of the solid as a function of dimensionless time, note the oscillation per the discussion above.

The geological implications of these types of coupled solidification/melting behavior are that some degree of fractionation can be attained by virtue of ascent and that repeated intrusion can yield spatial variations in melt fraction distribution in a manner that is difficult to generalize. This segregation may be important in the internal chemical organization of magma moving upward in the crust where material that crystallizes early on may or may not be remelted to yield cryptic spatial heterogeneities. This is also important in terms of understanding the heat transfer systematics attendant with plutonism where it is commonly assumed that efficient internal convection must occur as a means of reducing the ductility contrast between magma and country rock e.g. Mahon et al.(1988). As shown above, the heat transfer at the margins will depend on the solidification process which will inhibit efficient heat transfer to the surroundings. Thus, the isotherm that defines the region where yielding will occur will migrate into the intrusion and the magma body will then be intruding itself and telescoping upward into the crust leaving refractory phases and forming concentrically zoned suites. The net effect of this rheological partitioning is to induce a vertical stretching of magma bodies in the mid crust, below the brittle-ductile transition. Understanding this processes by examining the plutonic record is difficult as the strain field as given by rock fabrics can have contributions from a variety of processes (Paterson et al., 1989).

The thermal and mechanical perturbations that are ultimately manifested as volcanoes and plutons represent crustal scale processes (Chappell and Stephens, 1988; Hollister et al., 1989; Miller et al., 1988). Intermediate to evolved plutons in the mid-to-deep crust are testimony to the fact that petrogenetic processes need not all occur at 1kb (Marsh, 1989a) or require rapid cooling with vigorous convection. The demonstrably open system character of magmatism in every tectonic setting, and the striking diversity of chemical types in some volcanic fields, suggests that magmas are gregarious and that magma chambers are essentially thermal windows in the crust which act as manifolds (Hill et al., 1985). Magmatic systems assemble themselves from the bottom up, perhaps yielding a crustal scale thermal regime as pictured by Marsh (1984) and shown schematically in Figure 6, where the perturbation isotherms which delimit the region of mobilization open outward and downward.

Quantitative treatments of the crustal thermal regime during intrusion can be found in Marsh (1982), Singer et al. (1988), Mahon et al. (1988) and Mareschal and Bergantz (1990). One useful way to demonstrate the way in the initial geotherm, volume and periodicity of intrusion, and influence of the thermal regime at the lower boundary is to write the heat equation as a Green's function formulation where it is assumed that the heat transfer can be written as a linear equation (no temperature dependence in thermophysical properties or source term):

$$T(x,z,t) = \int\!\!\int G\left(\bar{r},t|\bar{r}',\tau\right)\Big|_{\tau=0} F\left(\bar{r}'\right) dz'\,dx' + \frac{\alpha}{k}\int\!\!\int\!\!\int G\left(\bar{r},t|\bar{r}',\tau\right) A g\left(\bar{r}',\tau\right) dz'\,dx'\,d\tau$$

$$-\frac{\alpha}{k}\int\!\!\int \frac{\partial G}{\partial z'}\Big|_{z=H} T_H\left(x',\tau\right) dx'\,d\tau \qquad (16)$$

where α is the thermal diffusivity, k conductivity, r a general spatial variable, $F(r)$ the initial geotherm, T_H the temperature at the bottom of the crust or region of underplating, $g(r,t)$ the thermal source term distribution for the sensible and latent associated with

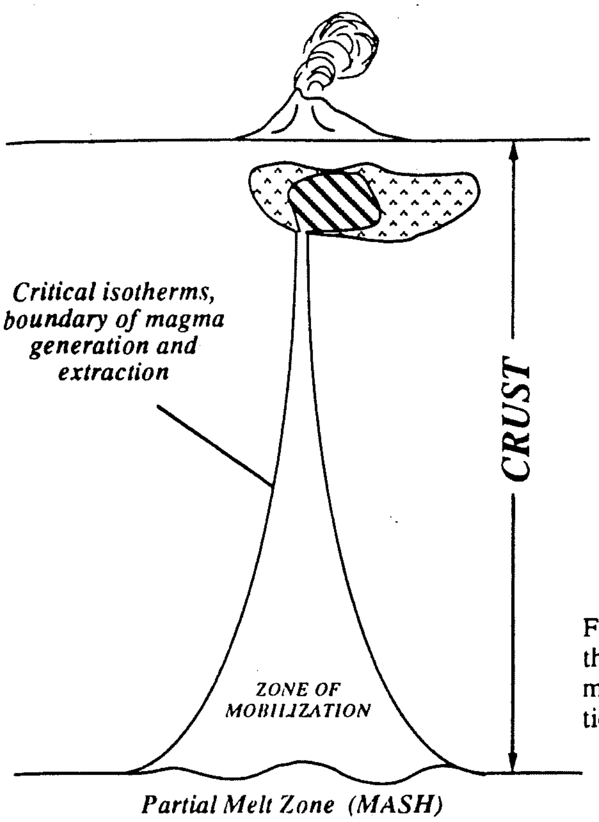

Figure 6. Schematic of crustal scale perturbation thermal field after Marsh (1984). Magmas that ultimately form shallow complexes can have contributions from a number of crustal levels.

intrusion and $G(r,t;r,\tau)$ the Green's function. The actual form of the Green's function will depend on the coordinate system, but should be understood to be the response of a homogeneous system with homogeneous boundary conditions to a impulse source. It is apparent from (16) that the temperature field in the crust can be written as the convolution of the Green's function with the initial geotherm, lower boundary temperature and thermal source term. The influence of crustal rheology and of melt fraction, as a function of lithology, will appear in the explicit form given for the source term, $g(r,t)$. It is this term which will determine the general morphology of the perturbation isotherms shown in Figure 6. In detail, the perturbation isotherms will not be smooth, that will depend on the form of $g(r,t)$ (Bergantz, in prep.) and on the intrusion to extrusion ratio, a quantity difficult to estimate. However, it is useful to inquire whether the MASH paradigm can be extended to a crustal scale by virtue of a thermal parameterization.

The melt fraction distribution plays a key role in quantifying magmatic processes at all scales. It provides a means of understanding how the magma chambers are partitioned in terms of the progress of solidification, rheology, and eligibility to undergo mixing and extraction. A wide variety of fluid structures have been proposed for magmas and there is no question they play a role in driving petrologic diversity. Experimental work to provide volume melt and solid phase fraction diagrams are urgently needed to further refine the detailed structures and their relevance to magmatic processes.

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